The Individual and the Market: Paul Samuelson on (Homothetic) Santa Claus Economics*

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I remember meeting with Tibor Scitovsky, in Washington, just before the war, at which he told me he was writing a paper on "Community Indifference Curves." I risked a new friendship by replying" "That's strange. Long ago I proved that community indifference curves are impossible – they don't exist. (Samuelson, 1956, p. 1073)[1]

0. Introduction

Paul Samuelson often used the term "Santa Claus economics" for mathematical models with extremely strong and empirically unrealistic assumptions. Examples of Santa Claus models ranged widely from investment-gambling models involving logarithmic utility (Samuelson 1971), to Ramsey growth models (Samuelson 1983), to models using Sraffa's standard commodity (Samuelson 1987). Although Santa Claus models represented a broad class for Samuelson – some useful and some not – I will focus on one particular member of the Santa Claus family that he was very skeptical about: what he called homothetic general equilibrium models (where all agents have identical homothetic preferences). I will argue that Samuelson's concerns about these models provide important insights into how he viewed the relationship between the individual and the market: a relationship that has implications for not only his demand and general equilibrium theorizing, but also his broader political-economic vision.

The paper is arranged as follows. The first section is a general discussion of Samuelson's use of, and his commentary on, homothetic Santa Claus models; it will draw out some implications of this modeling strategy and relate them to his use of the representative agent, his views on welfare economics, as well as a number of other topics. Rather than including the relevant mathematical results in the main body of the text, these results and associated citations are summarized in an Appendix at the end of the paper. The second section explains why this particular aspect of Samuelson's work is important: both to the history of modern economics and to certain debates within contemporary economic theory.

[1] All page references to Samuelson's works reprinted in the seven volumes of The Collected Scientific Papers of Paul A. Samuelson will be to the reprinted versions.

1. Paul Samuelson and Homothetic Santa Claus Economics: Aggregation, Revealed (Market) Preference, and the Representative Agent

This paper will focus primarily on the Walrasian general equilibrium models of the third quarter of the twentieth century.[2] They were generally n-good perfectly competitive models with a demand and supply (and thus excess demand) function for each good. Consumers/traders were assumed to maximize well-ordered preferences subject to a linear budget constraint. In the pure exchange case the supply of the various goods was simply the sum of the initial endowments of the individual traders, and in the production case it came from profit-maximizing perfectly competitive firms. An extensive literature developed to analyze the various properties of such Walrasian models and the theory also served as the foundation for applied analysis in fields such as international trade, welfare economics, and public finance.

Although the profession's best human resources and sophisticated mathematical machinery were brought to bear on this Walrasian program during the period 1950-1975, the results proved to be very disappointing. The existence theorem and the fundamental theorems of welfare economics were of course very important results, but the majority of the topics that commanded the attention of those working within the program proved to be quite illusive. The desired results were never achieved for tâtonnement stability, uniqueness, qualitative comparative statics, and several other areas.[4] Results were obtained for special case after special case, but seldom for those that were empirically relevant and never for the general case. As Kenneth Arrow and Frank Hahn noted regarding stability: "There is a distressingly anecdotal air about our investigation; case succeeds case, but it was not found possible to lay down any general principles" (1971, p. 321). The problem was compounded of course by the Sonnenschein-Mantel-Debreu (hereafter SMD) results in the early 1970s which demonstrated that individual utility maximization placed almost no restrictions on market excess demand functions and thereby opened the door to a wide range of counterexamples to stability and uniqueness.[5] The methodological presumption of the Walrasian framework was that market-level restrictions should come from the behavior of individual economic agents, but the standard assumptions provided almost no market-level restrictions.

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[2] The literature is far to extensive to attempt to cite. This said, Arrow and Hahn (1971) is the canonical text.

[4] Again the literature is extensive but Arrow and Hahn (1971) is probably the single best source: see Chs. 11 and 12 on stability, Ch. 9 on uniqueness, Ch. 10 on comparative statics, and Chs. 6, 8, 13, and 14 on other popular areas of research.

[5] Debreu (1974), Mantel (1974, 1977), and Sonnenschein (1972, 1973). See Shafer and Sonnenschein 1982) for a survey and Rizvi (1990, 2006) for historical discussion.

In a number of different theoretical contexts Samuelson introduced a Walrasian general equilibrium model that did have sufficient structure at the agent level to be able to say very specific (and desirable) things about the market-level results generated by the competitive interaction of such agents. The model was the homothetic Santa Claus case of uniform homothetic tastes: where all economic agents have identical homothetic preferences (particularly Samuelson 1956, 1968, 1972, 1977, 1978, 1983).[6] As he described it:

> Thus consider the theoretically interesting, but alas empirically untrue, case where the consumer facing given prices always divides his expenditure among goods in the same way at low as he does at high incomes. In this special case of unitary income elasticities, doubling income doubles all the quantities. The indifference surfaces are known to be "homothetic," i.e., each surface is just a radial blowup of any other surface. And it is intuitively obvious that there is a way of numbering these indifference surfaces so that the indicator of utility … can be written as a homogeneous function of degree one in quantities, doubling when all quantities double. (1965, 74-75)

The restriction of identical homothetic preferences has very strong implications for the resulting general equilibrium of market demand and supply. Specifically, the uniform homothetic case has the following implications for market demand and general equilibrium (see the mathematical appendix for more discussion):

1) Aggregation: Market demands are a function of total income.
2) Market Rationality: Market demand functions exhibit the same properties – Slutsky and revealed preference – as individual demand functions.
3) Representative Agent: There exists a representative agent: the economy acts as if market demand were generated by a single representative consumer with well-ordered preferences (social indifference curves exist).
4) Welfare: Market demand functions have welfare implications.

[6] The paper that Samuelson himself referred to as the first example of the homothetic Santa Claus case was his 1956 paper on "Social Indifference Curves." This paper does present the model, but he called it the "Robinson Crusoe" case in this early paper and began calling it Santa Claus later. It is important to note that Samuelson's discussion of this class of models appeared in different contexts off and on throughout his career; he never changed his position, but neither did he ever dedicate an entire paper to the topic. As a result, the Samuelson citations I use will come from papers on a number of different topics and spread out over decades. As discussed in the appendix, the main results on homothetic preferences were Eisenberg (1961) and Gorman (1953), but some of the ideas can be found as far back as Antonelli (1886).

In addition to these properties of market (and excess) demand functions, the homothetic case also has strong implications for the general equilibrium price vector including uniqueness and stability (again see appendix for details). It is a very special case in which the general equilibrium of the entire competitive system exhibits the same behavior as the individual neoclassical agent. Unlike the general case where Walrasian equilibrium simply means demand = supply for each good, this is a system where the equilibrium exhibits rationality: the properties of a single representative agent maximizing a utility function. The market behaves as if it were a neoclassical agent and equilibrium is nothing more than the optimizing behavior of that representative agent.

The homothetic case provides such strong results compared to the general model because it eliminates all of the problems caused by the heterogeneity of agents and income effects. As Samuelson explains:

> My uniform-homothetic case is manageable in general equilibrium form because all income effects are nicely balanced out. As soon as we permit different individuals to have different tastes and factor endowments, income effects enormously complicate the general equilibrium analysis. (Samuelson, 1968, p. 66)

As Hahn later noted about such models:

> If the rest of economic theory proceeded on these assumptions, welfare economics, for instance, would become extremely simple and stability analysis, would be child's play. Indeed a competitive economy could always be studied as if it were maximizing a utility function. (Hahn, 1983a, p. 42)[7]

Samuelson noted that this system has the properties of both William Stanley Jevons's concept of a "trading body" (Samuelson, 1956, p. 1077) and the "group mind" he associated with Arthur Pigou's welfare economics (Samuelson, 1950a, p. 1069). He also remarked that it was the characterization of general equilibrium implicit in the literature on the socialist calculation debate:

> Classroom expositions often use Santa Claus cases like this one to make the portrayal of complicated general equilibrium look simple. A century ago, Friedrich von Wieser's Natural Value improved on the Crusoe metaphor by using the metaphor of the perfect communist state. This led, via V.

[7] And more critically he adds: "Much of what we have regarded as interesting and important would be lost" (ibid.).

> Pareto, E. Barone, L. von Mises, F. Taylor, A. P. Lerner, O. Lange, F. von Hayek, A. Bergson, and L. Kantorovitch, to fruitful debate on the role of market pricing in socialist planning. (Samuelson, 1983, pp. 224-25)

Since Samuelson is (rightly) associated with the revealed preference approach to consumer choice theory it is useful to emphasize the revealed preference interpretation of market rationality and the representative agent. Outside of the two good case Samuelson's original (1938a) "weak axiom of revealed preference" (WARP) was not sufficient for rationalization of demand: the weak axiom alone did not guarantee the integrability condition necessary for the demand function to be treated as if it were generated by a budget-constrained utility maximizing agent.[8] But later developments in revealed preference theory (Houthakker 1950, Samuelson 1950b) strengthened the revealed preference axiom to the "strong axiom of revealed preference" (SARP) which was equivalent to utility maximization. Of course if the strong axiom of revealed preference axiom holds on market (rather than individual) demand functions, then there always exists a rationalizing representative agent: i.e. the so-called Wald case where the market reflects "revealed group preference" (Samuelson, 1956, p. 1088).[9] Although Samuelson considered the revealed preference axiom holding on market demand functions as equivalent to the homothetic Santa Claus case, one can also think of it as an alternative way of getting to the representative agent result – and the associated general equilibrium properties – by circumventing the individual agents entirely. If the market demand functions satisfy SARP then they could have been generated by a representative agent maximizing a well-behaved utility function and the competitive equilibrium of such an economy will have all the nice properties of the homothetic Santa Claus case. This is an extreme version of the Santa Claus case: one that does without any individual agents at all.[10]

Samuelson introduced the homothetic Santa Claus case in a number of papers and explored the various implications for demand and general equilibrium, but what did he say about the appropriateness of such models? The short answer is that he was generally quite critical. First there is the repeated assertion that it is a very special case that is

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[8] See Hurwicz (1971) and Hurwicz and Uzawa (1971) for the technical results on integrability and Hands (2006, 2011a) for historical discussion.

[9] Since Abraham Wald (1951) was the first person to formally impose revealed preference assumptions on market demand or excess demand functions, such conditions were often called Wald's restrictions in the literature of the period.

[10] It is useful to note that the later literature (Varian 1983) has developed a homothetic version of revealed preference – the homothetic axiom of revealed preference (HARP) – that guarantees that demand function can be rationalized by homothetic preferences.

"unrealistic" (1956, p. 1077) and "empirically untrue" (1968, p. 74).[11] But given that all abstract mathematical models rest on empirically unrealistic assumptions, it is more significant that he was quite critical of applying revealed preference axioms directly to market demand functions. As Samuelson noted in 1955:

> Such an axiom holds for a single individual, but it is arbitrary to assume it holds for the market totals. Many plausible examples can be given of this fact. (Samuelson, 1955, pp. 499-500)

And repeated in more detail in Dorfman, Samuelson, and Solow in 1958:

> Why is this [revealed preference] assumption peculiar? Because the demand functions … are market demand functions, not individual demand functions. "Rationality" cannot be required of market demand functions because changes in prices normally change the distribution of income. With a changed income distribution, <u>different</u> "preferences" will be revealed. (Dorfman, Samuelson and Solow, 1958, p. 368)

As noted above, if the SARP holds on market demand functions then the demand functions behave as if they were generated by a representative agent and the Walrasian general equilibrium of the whole economy reduces to the consumer's equilibrium of that agent. Samuelson argued against such models on empirical, theoretical and political-economic grounds. The empirical reasons are clear from the two quotes above: in general, changes in prices will change the distribution of income which in turn will change the choices (i.e. demands) of the various agents. Both the homothetic Santa Claus case and simply assuming SARP on market demand functions eliminates this empirically important possibility. His other concerns are a bit more subtle.

If the equilibrium of the economy is identical with the utility maximization of a representative agent then the market equilibrium of demand = supply is the solution to agent's maximization problem. In the language of <u>Foundations</u> (1947), this is an example of "convertibility into a maximum problem" (p. 52). Samuelson noted that although it is sometimes possible to convert "a problem whose economic context does

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[11] The empirical inadequacy of the assumption of homotheticity is a common feature of general equilibrium theory during this period. For example Arrow and Hahn:

> It is apparent that the restrictions involved are pretty stringent. It is hard to believe not only that all households are alike in this sense, but also that all Engel curves pass through the origin so that as an individual gets poorer and poorer he nonetheless continues to mix holidays, say, in the same ratio to bread, as he did when he was richer. (1971, p. 220)

not suggest any human, purposive, maximizing behavior into a maximization problem," he was also clear that such conversion "is to be regarded as merely a technical device for the purpose of quickly developing the properties of that equilibrium position" (p. 53). Such conversions are sometimes possible, but one needs to be very careful. First, because it may not be empirically reasonable "since it will require fully as much knowledge to ascertain whether the conditions of a maximum position are met as would be necessary to answer the questions which might be asked" (p. 52),[12] but also for more normative reasons since "there is the danger that unwarranted teleological and normative welfare significance will be attributed to a position of equilibrium so defined" (ibid.). For Samuelson such teleology – assuming that markets behaved like neoclassical agents – harked back to the old-fashioned welfare economics where the invisible hand of the market brought about maximization of the sum of individual utilities: precisely the welfare economics that he had helped replace with the new welfare economics of the Bergson-Samuelson social welfare function (1947, Ch. 8). In the new welfare economics, competitive equilibrium is always associated with a Pareto optimal allocation, but there are many (in fact an infinite number of) Pareto allocations, and it takes the community's social welfare function to select a specific welfare maximizing point from this set. In the homothetic Santa Claus case the competitive equilibrium is the unique social welfare maximum (associated with the utility function of the representative agent) and this is a much stronger defense of the free market than Samuelson believed pure economic theory could, or should, provide. As he noted in the final paragraph of his Nobel lecture (quoting Herbert Davenport): "There is no reason why theoretical economics should be a monopoly of the reactionaries" (1970, p. 16).

Another, but related, way to see the tensions associated with the homothetic Santa Claus case is in Samuelson's analysis of the Walrasian tâtonnement price adjustment mechanism (Samuelson 1941, 1942, 1944, 1947). Samuelson's Foundations was dedicated to techniques for obtaining comparative statics results – meaningful theorems – but the book was divided into two separate parts: part one where the results were based on optimization and part two where they were based on dynamic stability conditions.[13] The second part was necessary because

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[12] As noted in the appendix, such conversion requires that certain symmetry conditions hold on the cross partial derivatives of the relevant functions and that is often not the case (or impossible to determine if it is the case).

[13] As Hahn summarized this aspect of Foundations:

> We owe it to Samuelson more than to anyone else that at the level of the individual agent, the maximization hypothesis has been fruitfully put to work … But our main interest here is the economy as a whole, or at least in market predictions. Here matters are less satisfactory … But when households are brought into the picture, then generally things fall apart unless we can abstract from income effects; that is, unless agents have identical homothetic utility functions (Gorman, 1953). In that rather special case, the economy can

there were many comparative statics questions that could not reasonably be converted into optimization problems; Keynesian macroeconomic models were of course one obvious case, but so were Walrasian general equilibrium models:

> There are many equilibrium systems encountered in economics which do not arise from an extremum problem and which cannot be converted into this form. The various simplified versions of the Keynesian system provide but a first example of what is after all the general case. A second example is that of the general equilibrium equations of Walras. (Samuelson, 1947, p. 138, emphasis in original)

Not only was the non-optimization-based part two of Foundations necessary for comparative statics in Walrasian models, it began with Chapter nine on "The Stability of Equilibrium: Comparative Statics and Dynamics" (also Samuelson 1941) where Samuelson specifically criticized the Walrasian multi-market stability analysis of John Hicks in Value and Capital (1946) precisely because it would only coincide with "true dynamic stability" when the cross partial derivatives of market excess demand functions were symmetric with respect to price (Samuelson, 1947, pp. 269-77).[14] Such symmetry (or integrability) is precisely the condition that the homothetic Santa Claus case (or simply assuming SARP on market demands) imposes on market excess demand functions. If the representative agent of the Santa Claus case – or assuming SARP on market demand functions – were appropriate in Walrasian economics then Samuelson would have no reason to criticize Hicks and/or propose his own stability condition; in fact, there would be no need for much of part two of Foundations. Of course this is precisely Samuelson's point in the above quotes about imposing revealed preference conditions on market demand functions. Market demand functions should be based on the utility maximizing behavior of potentially heterogeneous agents – not the Santa Claus case of a representative agent – and in such a case a separate set of tools for comparative statics analysis will be necessary (the tools that part two of Foundations provides). The inter-agent stability of the Walrasian price adjustment mechanism should not be reduced to the intra-agent stability of a utility-maximizing individual; the "Hicks procedure is clearly wrong" (ibid., p. 273) and the homothetic Santa Claus case is inappropriate in Walrasian general equilibrium theory.

> be viewed as if a single maximizing household were involved, and so the maximization hypothesis can deliver at full power. In general however, the equilibrium cannot be converted into the solution of an as if maximization. (Hahn, 1983b, p. 34)

[14] "Not working with an explicit dynamical model, Professor Hicks probably argued by analogy from well-known maximum conditions, whereby a maximum must hold for arbitrary displacements and through any transformation of variables." (Samuelson, 1947, p. 273)

In closing this section I would like to highlight Samuelson's concerns regarding the Santa Claus case by briefly discussing two examples where he did seem to be comfortable putting the representative agent to work in general equilibrium theory. Understanding specific cases where he considered it to be an appropriate modeling strategy helps us understand why he thought it was an inappropriate approach to demand and general equilibrium theory in general. One place where he seemed to be comfortable using the assumption of market rationality was in his original 1956 paper on "Social Indifference Curves." In that paper he not only defended the possibility of social indifference curves, but argued that it was the "first justification of the Wald hypothesis that market totals satisfy the 'weak axiom' of individual preference" (p. 1093). The reason this is not a general endorsement of the Santa Claus case is that this is a very specific model involving an optimal/ethical reallocation of income prior to the equilibrium of the competitive market. It is, as Samuelson noted, "more like welfare economics than like positive demand analysis" (ibid., p. 1082). The dangers of bringing in normative judgments are not a problem in such a model since it began with an initial social welfare function and stayed within its normative strictures throughout. There is no fear of inappropriate welfare implications when one starts with what Samuelson thought were appropriate welfare implications (a Bergson-Samuelson social welfare function). The second case is his 1978 paper on "Pseudo-Maximization" (Samuelson 1978). This is a case where he employed a representative firm to discuss market factor demand curves: where the "industry's comparative statics can be shown to behave as if the industry had a soul and an integrable mind" (Samuelson, 1978, p. 89). This is clearly a case where he thought aggregation was appropriate, but notice that it exclusively involved production, not consumer choice, theory. First of all there are, as he says, "no normative or welfare connotations implied by this mechanical construction" (ibid., p. 102) and secondly, there are none of the problems associated with income effects in production theory.[17] The representative firm may be appropriate in this case, because it raises none of the issues that Samuelson found so problematic about the homothetic Santa Claus case in consumer demand theory.

To summarize the lesson of this section, Samuelson's writings on the homothetic Santa Claus case and his related work make it clear that at least in the case of demand theory and Walrasian general equilibrium, Samuelson was quite critical of treating competitive markets like individual agents. There were several reasons the Santa Claus case was problematic: 1) the assumptions associated with such models were empirically inappropriate (distribution and income effects matter), 2) he was critical of applying revealed preference theory directly to market

[17] The cross partial derivatives of factor demand functions are always symmetric (one of the important additional restrictions that homotheticity imposes on market demand functions for consumer goods).

demand functions, 3) market demand functions were not one of the cases where an equilibrium system could be converted into a extremum problem, 4) the symmetry associated with imposing the homotheticity assumption on excess demand functions nullifies all of Samuelson's influential work on the stability of competitive equilibrium, and 5) there were a number of normative and/or political-economic implications of reducing competitive markets to maximizing agents that Samuelson found disquieting. All of this said it is important to point out that Samuelson did not consider the representative agent to be inappropriate in all economic models. There were a number of theoretical contexts – growth theory, international trade theory, and financial economics, for example – where he seemed to be comfortable with the assumption of uniform homothetic tastes. But such examples only strengthen the argument that he was particularly uncomfortable with such assumptions in the context of consumer choice and Walrasian general equilibrium theory. If the problems he saw with the homothetic case were merely technical, he would have rejected it in all classes of economic models, but he didn't. His concerns were not technical; they were empirical, theoretical, and normative, and they concerned the specific case of Walrasian demand and general equilibrium modeling.

2. Why this Matters: In the History of Modern Economics and in Contemporary Economic Theory

In this section I will discuss why Samuelson's position on homothetic Santa Claus models matters: why it is relevant to our understanding of modern economics – particularly the differences between the Walrasian economics of the mid-twentieth century and recent Walrasian theory – and also why it matters to certain debates within contemporary economic theory (both macro and micro).

The term "neoclassical synthesis" has re-entered the macroeconomics literature during the last few years (Goodfriend and King 1997, Woodford 2003) as a label for a dynamic stochastic general equilibrium (DSGE) modeling strategy with roots in the Real Business Cycle and New Keynesian literature of the end of the twentieth century. One of the defining features of this literature is the use of the representative agent. Unlike the Walrasian general equilibrium theorizing discussed above, where the models explicitly involved potentially heterogeneous economic agents, these models generally employ a single representative agent where the equilibrium of the economic system reduces to the optimizing behavior of that agent: equilibrium conditions

are simply first order conditions.[18] Gone is the earlier motivation for Walrasian general equilibrium theory: explaining how the competitive price system coordinates the actions of self-interested, but potentially dissimilar, economic agents. Gone is the Walrasian vision of Arrow and Hahn:

> Whatever the source of the concept, the notion that a social system moved by independent actions in pursuit of different values is consistent with a final coherent state of balance, and one in which the outcomes may be quite different from those intended by the agents, is surely the most important intellectual contribution that economic thought has made to the general understanding of social processes. (1971, p. 1)

The stated motivation for the elimination of different values and unintended consequences from Walrasian economics is usually that i) it avoids the ad-hocness problems of earlier Keynesian models identified by Robert Lucas (1976, 1989) and others, and ii) the utility function of the representative agent "provides a natural objective in terms of which alternative policies should be evaluated" (Woodford, 2003, p. 12), but there is seldom recognition of the Santa Claus properties of such models. This is an extremely strong methodological commitment – one quite at odds with the Walrasian theorizing of an earlier generation – and yet one that seems to be shared by modern macroeconomic theorists of both the new classical and new Keynesian perspectives. The program claims Walrasian microeconomic roots – and privileges such foundations – and yet, by use of the representative agent rejects the earlier Walrasian vision of a competitive economy and the traditional explandandum of such theorizing, but also turns its back on the research program's most important technical results: the SMD theorems. As Kevin Hoover explains:

> The representative-agent program elevates the claims of microeconomics in some version or other to the utmost importance, while at the same time not acknowledging that the very microeconomic theory it privileges undermines, in the guise of the Sonnenschein-Debreu-Mantel theorem, the direct analogue of a plausible utility function for an individual agent. Kirman's (1992) survey article on the representative agent, which highlights the lack of analogy, is well-cited; yet, it is striking that almost all of the citations

[18] There have of course been many recent attempts to introduce heterogeneous agents into DSGE models, but the representative agent is the traditional, and still standard, framework. See Athreya (2013) for a recent discussion of this literature and the key references.

> are by critics of the representative-agent program; there is little evidence that advocates have even noticed the argument against their approach. (Hoover, 2012, p. 51)

DSGE macroeconomics does not simply require microfoundations, or even Walrasian microfoundations, it employs a very specific and in many ways quite non-Walrasian representative agent microfoundations and it seems to do so without explicit recognition of the assumptions that are being made, the sharp break with the earlier Walrasian tradition, or the potential pitfalls of this strategy; it is as if "the new classical macroeconomists sleepwalked into their most characteristic methodological position" (Hoover, 2012, p. 50). Finally, the representative agent has also been the source of much criticism of the relevance of DSGE models to the recent financial crisis: "Many of the shortcomings of macroeconomic modeling that have been raised by critics in relationship to the recent financial crisis – the lack of focus on heterogeneity and coordination problems, the inadequacy of the standard representation of the financial system, the failure to address the possibility of systemic market failure – are related, in part, to the particular ways in which mainstream macroeconomics has attempted to provide microfoundations" (Durate and Lima, 2012, p. 13).

All of these are important and interesting questions for modern economics – theory and practice – but what specific historical questions and issues are raised by the above examination of the homothetic Santa Claus case? There are many, but I will note just two interrelated issues.

First, recognition of the Samuelson's position on homothetic general equilibrium models helps us understand a significant and unrecognized difference between the old and new version of the neoclassical synthesis. As noted, it is common to argue that the main differences are that unlike the old, the new neoclassical synthesis emphasizes Walrasian microfoundations and rejected the Keynesian theoretical framework, and to some extent, this is certainly correct. But these differences do not exhaust the differences between the earlier and contemporary versions of the neoclassical synthesis. Through the use of the representative agent, the new synthesis eliminates the traditional Walrasian difference between equilibrium (demand = supply) and the optimization of an individual economic agent. This distinction stabilized in the literature of the decades immediately following the publication of Samuelson's Foundations (Hands 2010), but now seems to be blurred. Three separate types of economic theorizing were involved in the first neoclassical synthesis: the individual choice behavior of the economic agent; the competitive market with prices adjusting at various speeds toward general equilibrium; and the behavior, adjustment, and equilibrium values of aggregate variables such as consumption and investment. In the modern neoclassical synthesis all three of these aspects are reduced to the behavior of the neoclassical agent.

Second, Samuelson's skepticism regarding the Santa Claus case, helps us understand the historical evolution of the Walrasian research program. The Walrasian perspective that dominated the middle of the twentieth century viewed the price system in a competitive market as coordinating the actions of potentially dissimilar economic agents. As Arrow and Hahn noted: the role of different values and unintended consequences in competitive markets. Alternatively, the microfoundations of the contemporary neoclassical synthesis are based on a group mind where a single set of preferences reigns and nothing is unintended.

One of the many issues is that it is not at all clear how this happened. How did the Walrasian vision change so significantly and seemingly effortlessly? Why did so many of the sacrosanct priorities of mid-century Walrasian theorizing – such as a commitment to the heterogeneity of agents, the difference between equilibrium and optimization, the idea of prices adjusting to equilibrium, etc. – all cease to be relevant just a few decades later. There is, and will be, of course no simple answer. It is often argued that the representative agent was a way of avoiding the negative implications of the SMD results (Kirman 1992) -- and there is no doubt that it does serve this purpose – but as noted in the quote from Hoover above, there does not even seem to be any recognition that there was a SMD problem, much less a self-conscious effort to circumvent it, among those most responsible for the changes in Walrasian practice. It is also important to point out that during the same time that the representative agent was solidifying its grip within macroeconomics, the empirical adequacy of the utility maximizing agent came under serious attack by experimental psychologists, behavioral economists, neuroeconomists, and others, but one would think that such criticism would be a reason to question such maximization-based theorizing, not to promote it from the individual agent to the entire economy. These questions will certainly need additional historical investigation, but Samuelson's view of these issues is an important first step toward a better understanding this significant transformation of Walrasian economics.

The final part of my discussion turns from topics that have traditionally been considered macroeconomics to a topic traditionally considered to be microeconomics. The topic is revealed preference theory. Like the representative agent in macro, revealed preference theory has received a new lease on life during the last decade or so. Gone are the days when revealed preference was little more than a short, and often skipped, subsection in the consumer choice chapter of microeconomics textbooks. Building on the early work of Afriat (1967) and Diewert (1973) an extensive literature has developed that applies revealed preference theory to finite choice data (Varian 1983, 1985) and it is now a serious challenger to more traditional econometric techniques in applied demand analysis. The success of these applied revealed preference techniques

has prompted a number of authors to make fairly strong methodological claims about how these techniques have replace, or should replace, all other approaches to choice-theoretic economics (Binmore 2009, Gul and Pesendorfer 2009).[19]

One of the important features of these empirical revealed preference techniques is that any price-quantity data consistent with the revealed preference axiom can be "rationalized" – it can be treated as if it were generated by a budget-constrained utility maximizing consumer. Of course, if the relevant price-quantity data is generated by an individual consumer then the rationalization involves the consumer's utility function, but suppose the price-quantity data is market data; in that case the utility function is that of the market. The rationalization is market rationalization and the utility function is that of a representative agent. This is exactly the application of revealed preference theory to market data that Samuelson thought was dangerous. Some defenders of these empirical revealed preference techniques praise its potential application directly to market data as one of the main virtues of such approaches (Ross, 2011a, 2011b, 2012) – and whether such praise is warranted or not in particular empirical applications is a question for the relevant practitioners – but Samuelson is often cited as the inspiration for such an interpretation. For example Don Ross says:

> This dynamic internal to the discipline reflected felt pressure to make economics a social science, which could arrive at conclusions by studying aggregate statistics and could ignore idiosyncrasies of individual consumers. Choice in the folk sense, or in the refined sense of post-behaviourist psychology, simply has no role at all in the theory presented in the Foundations. That theory takes account of observable aggregate demand and if this has certain testable properties then the existence of continuous preference fields is implied. What stabilizes these fields might or might not be properties of individual psychologies; the revealed preference theorist disavows professional interest in this question, a point on which Samuelson is explicit. Gul and Pesendorfer merely reaffirm this venerable tradition, and pound the table on its behalf. (Ross, 2011a, p. 221)

This is of course not what Samuelson makes explicit. What he made explicit was that such exercises in Santa Claus revealed preference theory on market demand functions were highly questionable for a number of reasons.[20] Again there is an historical point here as there was

[19] See Hands (2011b, 2013) and Varian (2006) for discussions of these developments.

[20] It is also useful to point out that there is very little revealed preference theory, even for individual agents, in Foundations. The entire discussion of consumer choice theory is conducted in terms of standard ordinal

with the representative agent in DSGE models discussed above. The DSGE models are self-consciously Walrasian and yet employ theoretical constructions that are inconsistent with earlier practice within Walrasian economics, and here an argument is made in favor of applying revealed preference theory to market data based on Samuelson's endorsement even though he explicitly argued against such practices.[21] In both cases we can use Samuelson's discussion of the homothetic Santa Claus case to understand his position on these contemporary debates. Given what Samuelson had to say about this case, it seems very unlikely that he would support either of these recent theoretical moves: either DSGE or the rationalization of market data via revealed preference. This historical point need not involve a criticism of contemporary practice; it simply points out the tension between Samuelson's own interpretation of the representative agent and the way it is used in the recent literature (macro and micro). Awareness of this tension does not provide us with a complete understanding of how the transformation of Walrasian general equilibrium and/or revealed preference theory took place, but since recognizing differences is a necessary precondition to understanding them, it does provide an important first step toward understanding the various causal forces behind these transformations.

3. Conclusion

This paper has explained Samuelson's position on homothetic Santa Claus models in demand and general equilibrium theory. The characteristics of such models are explained – the technical results are in the appendix – as well as the various sources of his concern about such modeling strategies. It is important to point out that Samuelson was comfortable with the use of such constructs – particularly the representative agent – in a number of other types of economic models, which makes his concern over these particular applications particularly

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utility theory with one passing reference to Samuelson's original 1938 paper and the term "revealed preference" does not appear in the consumer choice chapter at all, but rather in his discussion of index numbers. See Hands (2014) for a discussion of Samuelson on revealed preference theory.

[21] In fairness to Ross, his historiography allows him to be perfectly comfortable with this fact. His approach is explicitly rational reconstruction. As he explains:

> It searches for tendencies in the development of scientific thought that were influenced by conceptual logic. It is frank about the fact that we are in a much better position to discern these tendencies and this logic than were the people who couldn't know where the story was going to go. Such history starts from where we now find ourselves and looks for anticipations in earlier problem settings and conceptual evolution – looks, that is, for the growing seeds of the present in the relatively chaotic past where they were difficult to distinguish from seeds that didn't germinate. (Ross, 2009, p. 101)

In a sense Ross' historiography is a version of the revealed preference theory he endorses – it is revealed preference historicism – as the "economic agent" choices can be rationalized, so can the choices of earlier economists. I believe this approach to the history of economic thought is problematic (Hands 2009), but it does make Samuelson's remarks irrelevant to Ross' reconstruction of the history.

interesting and important. Section two pointed out some historical tensions highlighted by Samuelson's interpretation of these particular homothetic Santa Claus models. These issues concerned important differences between the original neoclassical synthesis and the DSGE neoclassical synthesis of recent macroeconomics, the evolution of the Walrasian theoretical framework, and contemporary revealed preference theory.

Appendix: (Homothetic) Santa Claus General Equilibrium

The purpose of this appendix is to pull together many of the relevant theoretical results and to summarize them in a convenient way. To this end, I will make a number of simplifying assumptions. I will only consider pure exchange and assume that all functions are well-behaved and that prices are strictly positive. The pure exchange case is sufficient since the results can easily be extended to a production economy and also because all of the problematic issues in Walrasian general equilibrium theory – stability, uniqueness, etc. – are associated with demand, not supply (in particular, with the income effects on the demand side).

Individual and Market Demand and Excess Demand

Let the economy have H traders $h = 1,2,\ldots,H$ and n commodities x_i for $i = 1,2,\ldots,n$ with competitive prices $p = (p_1, p_2,\ldots, p_n) \in \Re_n^{++}$. The excess demand for each good i by trader h is given by

$$z_i^h(p, M^h) = x_i^h(p, M^h) - \omega_i^h ,$$

where $x_i^h(p, M^h)$ is the demand for, and ω_i^h the initial endowment of, good i by trader h, with $M^h = \sum_{i=1}^{n} p_i \omega_i^h$ for all h. Each demand function $x_i^h(p, M^h)$ satisfies the Slutsky equation:

$$\frac{\partial x_i^h}{\partial p_j} = S_{ij}^h - x_j^h \frac{\partial x_i^h}{\partial M^h},$$

where S_{ij}^h is the Slutsky Substitution term. The matrix of substitution terms S^h has the standard restrictions: i) negative diagonal elements ($S_{ii}^h < 0$ for all i), ii) is symmetric ($S_{ij}^h = S_{ji}^h$ for all $i \neq j$), and iii) is negative semi-definite ($x^T S^h x \leq 0$ for all $x \neq 0$).

The market excess demand for good i will be given by $z_i(p)$ where

$$z_i(p) = z_i(p, \omega^1, \omega^2, \ldots, \omega^H) = \sum_{h=1}^{H} z_i^h(p, \omega^h).$$

These excess demand functions are homogeneous of degree zero (H) in (unit of account) prices and satisfy Walras's Law (W),

$$z_i(p) = z_i(\lambda p) \text{ for all } \lambda > 0 \text{ and for all } i = 1, 2, \ldots, n, \qquad \text{(H)}$$

$$p^T z(p) = \sum_{i=1}^{n} p_i z_i(p) = 0. \qquad \text{(W)}$$

Under (H) we can normalize prices so that $p_n = 1$, and if we continue to use p for (now normalized) prices, we have $p = (p_i, p_2, \ldots, p_{n-1}, 1)$.

Given the standard existence theorems a competitive equilibrium price vector will always exist for this economy. The competitive equilibrium is given by:

$$z_i(p^*) = 0 \text{ for all } i = 1, 2, \ldots, n, \qquad (2)$$

where $p^* = (p_1^*, p_2^*, \ldots, p_{n-1}^*, 1)$.

The (Homothetic) Santa Claus Case

So far all of this is just standard mid-twentieth century Walrasian general equilibrium theory. Now let us add the Santa Claus assumption that all agents have identical homothetic preferences; "that all men have identical homothetic indifference contours, so that every dollar spent gets spent in the same way, no matter whose hands it is in" (Samuelson, 1938b, 1965 Postscript, p. 34). There are other assumptions that would give us similar results,[22] but I will only discuss this paradigmatic Santa Claus case.

A preference ordering is homothetic if an agent prefers x to y if and only if they prefer λx to λy for all λ>0 (or equivalently, that all indifference sets are radial blowups: that MRS(x_0) = MRS(λx_0) for any λ>0 where MRS = marginal rate of substitution). In terms of the properties of the utility function, U(x) represents homothetic preferences if it can be written as a

[22] For example: that all individuals have homothetic preferences (but not necessarily identical) and proportional income (a fixed distribution of income). See Chipman (1974, 2006) or Polemarchakis (1983). There are similar results involving quasi-linear preferences (Mas-Colell, Whinston, and Green, 1995, pp. 80-81).

monotonic transformation of a homogeneous function (if there exists a homogeneous function F, and a function G with G'>0, such that U(x)=G[F(x)]).[23] When preferences are homothetic the individual demand functions $x_i^h(p, M^h)$ will have a number of important properties (Chipman 1974) including:

a) being homogeneous of degree 1 in income:
$x_i^h(p, \lambda M^h) = \lambda x_i^h(p, M^h)$ for all λ>0.

b) being unit income elastic: $\left(\frac{M^h}{x_i^h}\right)\left(\frac{\partial x_i^h}{\partial M^h}\right) = 1$.

c) and having symmetric cross partial derivatives with respect to price:[24] $\frac{\partial x_i^h}{\partial p_j} = \frac{\partial x_j^h}{\partial p_i}$ for all $i \neq j$.

Aggregation

Aggregation means many things in modern economics,[25] but here we are only interested in aggregation of individual demand functions.[26] Even so, there are many ways of thinking about individual-to-market demand aggregation. One question concerns the properties of market demand generated by adding up individual demand functions, but alternatively there is the question of whether a market demand could have been generated by the budget-constrained utility maximization of (one or many) individual consumers. The individual-to-market demand aggregation question suggests (at least) the following four questions.[27]

A1) Demand Function Aggregation: Can the market demand functions be written as a function of total income? In other words is it the case that

[23] Since demand functions are invariant with respect to any monotonic transformation, the results for homothetic preferences/utility could just as well be obtained from homogeneous functions (as some authors do).
[24] Note this is symmetry on the regular demand functions, not just symmetry of the Slutsky terms which holds for any demand function. Such symmetry was termed "Hotelling Symmetry" in Chipman and Moore (1976) because of its use in Hotelling (1932).
[25] See Hoover (2012) for a recent discussion.
[26] Even when the concern is demand function aggregation there are empirical as well as theoretical issues. The empirical issues concern the circumstances under which it is empirically appropriate to use total income (often available) to estimate market demand functions rather than the income levels of all of the individuals in the market (seldom available). The empirical and theoretical issues are of course related, but I will focus exclusively on the latter.
[27] This list is a modified version of the list in Mas-Colell, Whinston, and Green (1995, p. 105).

$$x_i(p,M) = x_i(p,M^1,M^2,\ldots,M^H) = \sum_{h=1}^{H} x_i^h(p,M^h) \text{ for all i?}$$

A2) Market Demand Rationality: Does market demand function $x_i(p,M)$ exhibit the same properties as the individual demand functions x_i^h. This question has two forms:

A2a) Ordinal Utility Version: Does the market Slutsky Substitution matrix with representative element

$$\frac{\partial x_i}{\partial p_j} = S_{ij} - x_j \frac{\partial x_i}{\partial M},$$

exhibit the properties $S_{ii} < 0$ for all i, $S_{ij} = S_{ji}$ for all $i \neq j$, and $x^T S\, x \leq 0$ for all $x \neq 0$.

A2b) Revealed Preference Version: Does the market demand function exhibit the Strong Axiom of revealed preference (SARP)?[28]

A3) Representative Consumer: Does the economy behave as if the market demand function were generated by a representative consumer maximizing a well-behaved ordinal utility function subject to the constraint of aggregate income M?

A4) Market Welfare Implications: Do the market demand functions have welfare significance?

In general of course, even in pure exchange, none of these questions can be answered in the affirmative. In the general case where traders have potentially different preferences, market demand will depend on each of the $M^h s$ and not just total M and therefore neither A2a or A2b will hold on market demand. This in turn implies that market demand need not behave as if it were generated by a representative consumer and it also does not necessarily have any particular welfare implications.

The most important message of this appendix is that in the Santa Claus case where all individuals have identical homothetic preferences, all of these questions can be answered affirmatively. The various papers that demonstrate these results are interrelated. (A1) and (A3) were demonstrated in Chipman (1974), but versions of the result go back to Eisenberg (1961), Gorman (1953) and even Antonelli (1886). Also see

[28] Or other forms of revealed preference such as the Weak Axiom (WARP) or Generalized Axiom (GARP).

Chipman (2006) for (A1) and Arrow and Hahn (1971, p. 220) for (A3). Once we have (A3) and the economy's market demand functions reflect the choices of a single utility maximizing representative consumer, then both (A2a) and (A2b) follow from standard results in demand theory.

The question in (A4) can also be answered affirmatively, but is more complex than (A1)-(A3). Intuitively if the economy is acting as if it were maximizing the utility of the representative agent, then the agent's utility function would be a natural choice for the social welfare function. The problem is that we would like to have results that hook up more directly with the way that welfare is discussed in modern economics: measuring welfare from potentially observable demand functions and being able to accommodate a broader class of (i.e. any Bergson-Samuelson) social welfare functions. John Chipman and James Moore provided a number of results for the former problem – for example, Chipman and Moore (1976, 1980a) demonstrated that in the Santa Claus case Marshallian consumer surplus is an appropriate welfare measure and Chipman and Moore (1973, 1980b) obtained a similar result for changes in real national income[29] – and Samuelson (1956) provided the initial results on the latter problem. Samuelson (1956) approached the question through the analog of family demand. A family generates demand functions, but they reflect neither the preferences of a single agent nor the mere sum of the choices of individual family members. The key is that redistribution takes place within the household before the demand functions are generated, and as a result the demand functions will depend on that (assumed ethically optimal) intra-household allocation: "since blood is thicker than water, the preferences of the different members are interrelated by what might be called a 'consensus' or 'social welfare function' which takes into account the deservingness or ethical worth of the consumption levels of each of the members" (Samuelson, 1956, p. 1082). Translating this into the wider economic context, Samuelson's model assumed that "the government optimally redistributes income so as to maximize a Bergson-Samuelson social-welfare function .. this requires the government to implement an *optimal-income-distribution* function" (Chipman, 2006, p. 114). The Santa Claus case of identical homothetic preferences conveniently provides one such distribution function (Chipman 1974, 2006; Mas-Colell, Whinston, and Green, 1995, pp. 116-121).

General Equilibrium

The power of the homothetic Santa Claus case is most apparent when it is applied to traditional questions in Walrasian general equilibrium theory such as stability and uniqueness. It allows us to "show how simple general equilibrium can become if every dollar is spent

[29] Also see Rader (1976), Samuelson (1990), and Silberberg (1972).

in the same way by people who have uniform homothetic tastes" (Samuelson, 1983, p. 226).

Uniqueness: As noted above, the standard existence theorems guarantee that a general equilibrium price vector p^* will always exist, but in the homothetic Santa Claus case we have the much stronger result that it will also be unique (Arrow and Hahn, 1971, p. 221). There are many reasons this should not be surprising. First, intuitively if the economy behaves as if there is just one consumer and that consumer has a well-behaved utility function it seems natural that the uniqueness of the solution to the (sole) consumer choice problem should carry over to the uniqueness of the competitive equilibrium price vector. Second, and more technically, the first formal proof of uniqueness in a Walrasian general equilibrium model (Wald 1951) assumed that market excess demand functions satisfied a revealed preference condition – specifically $p^* z(p) > 0$ for all $p \neq p^*$ which, in strong form, is equivalent to the economy acting like there is a single representative agent (A2b).[30]

Stability: Most of the stability results for Walrasian general equilibrium theory from this period were based on the differential equation version of the Walrasian tâtonnement price adjustment mechanism popularized by Samuelson in (1941, 1942, 1944) and reproduced as chapters nine and ten in Foundations (1947):

$$\frac{dp_i}{dt} = z_i(p) \text{ for all i.} \qquad \text{(T)}$$

There were many other versions of this system of differential equations, involving speeds of adjustment and such, but they all shared the same basic property that positive excess demand for a particular good meant that its price would increase, while negative excess demand meant that it would decrease. This framework was used to analyze both local stability (non-equilibrium price vectors within an epsilon neighborhood of p*) and global stability (non-equilibrium price vectors throughout the price domain); most of the local results were obtained by imposing restrictions on the market excess demand Jacobian at p* while the global results – following Arrow and Hurwicz (1958) and Arrow, Block, and Hurwicz (1959) – involved the use of a Lyapunov function (see Chs. 11 and 12 of Arrow and Hahn 1971).

[30] In fact the most intensive discussion of uniqueness in Walrasian general equilibrium theory – Chapter nine of Arrow and Hahn (1971) – is all framed in terms of this revealed preference condition. The authors discuss a number of conditions that are sufficient for uniqueness, but in almost every cases they are conditions that are sufficient for Wald's revealed preference axiom that technically does all of the heavy lifting.

The homothetic Santa Claus case discussed here is globally stable (Arrow and Hahn, 1971, p. 287).[31] There are at least two different ways to get this result. Arrow and Hahn make the argument by using the utility function of the representative consumer as a Lyapunov function, in particular, the adjustment mechanism (T) becomes a gradient process for minimizing [U(x(p)) – U(x(p*))] (p. 289).[32] An alternative approach is to use the sum of the squares of the excess demand functions as a Lyapunov function and then exploit the property that in this case the excess demand Jacobian is symmetric (Arrow and Hurwicz 1958, p. 536).

The bottom line is that the homothetic Santa Claus case is extremely well-behaved and has all of the most desirable properties in Walrasian general equilibrium systems. It is indeed a Santa Claus model.[33]

[31] Arrow and Hahn call this case – the case of the one household economy – the "Hicksian" case (p. 220) – and they demonstrate that the Hicksian case is globally, and independently locally, stable.

[32] Notice that while this is true for the homothetic Santa Claus (Hicksian) case it is certainly not true in general: "If the auctioneer's rule may be treated as an intelligent method of maximizing or minimizing some relevant function, then if such a function is well behaved … we should expect the rule to exhibit global stability. Unfortunately, however, except in exceptional circumstances of which the Hicksian is one instance, the price mechanism cannot be taken to act as if someone were trying to maximize or minimize some well-behaved function of prices" (Arrow and Hahn, 1971, p. 278).

[33] The general equilibrium model discussed in Mirowski and Hands (1998) has similar properties, but starts from a different (non-budget-constrained) characterization of the individual choice problem.

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