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Stabilizing Consumer Choice: The Role of "True Dynamic Stability" and Related Concepts in the History of Consumer Choice Theory*

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Abstract: It is often argued that the inability of Arrow-Debreu general equilibrium theory to produce an adequate proof of the stability of the Walrasian price adjustment mechanism was one of the program's most significant failures. This paper will not question this standard interpretation of the history of general equilibrium theory, but makes the case that characterizing the "stability" question in terms of market stability – in particular the stability of the equilibrium price vector in Walrasian general equilibrium model – actually helped to stabilize the standard model of consumer choice in general equilibrium theory and elsewhere within microeconomics. The problem of the stability of "consumer's equilibrium" was much discussed early in the twentieth century and it has recently re-emerged in a different guise as the "endowment effects" and "reference dependencies" of contemporary behavioral economics, and yet it disappeared from mainstream discussion during the period 1950-1980. This paper argues that shifting the discussion from the intra-agent stability of the individual consumer to the inter-agent stability of the competitive market contributed – despite its ultimately negative impact on general equilibrium theory – to the long period of stable normal science consumer choice theory enjoyed during the middle of the 20th century.

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0. Introduction

This paper is motivated by, and will attempt to reconcile, the following four features of twentieth-century microeconomic theory (some of these are well-known and well-accepted, while others are less so).

1. Paul Samuelson's early papers on "true dynamic stability" (1941, 1942) played a key role in stabilizing the way economic theorists thought about, wrote about, and formalized, the concepts of "stable" and "unstable" equilibrium during the period 1950-1980. Although Samuelson focused exclusively on local stability, his basic conceptual framework was also adopted by those working on the global stability of competitive equilibrium (Arrow and Hurwicz 1958; Arrow, Block and Hurwicz 1959; McKenzie 1960; Newman 1961; Uzawa 1961 and others) as well as by most analyzing stability in other classes of economic models (macroeconomics, international trade theory, etc.) during this period.
2. Many of the criticisms of rational choice theory recently raised within behavioral economics, experimental economics, and experimental psychology – endowment effects, reference dependence, framing, threshold effects, etc. (see Camerer and Loewenstein 2004; Kahneman and Tversky 2000; Knetsch 1989; Rabin 1998, 2004; and Thaler 1980 for example) – were discussed in neoclassical choice theory during the early twentieth century under the broad heading of the "integrability problem," but later disappeared from the literature after integrability was redefined as "rationalizing" individual demand functions (and under this definition, formally solved).
3. The late 1940s – particularly Samuelson's <u>Foundations</u> (1947) – represented a fairly sharp break in the way that consumer choice theory was presented in mathematical economics. Prior to this period the model of the consumer maximizing a well-behaved ordinal utility function subject to a linear budget constraint, was presented in relatively dynamic terms; the "consumer's equilibrium" was a position that the agent "moved toward" and then "stopped at" once the optimum was reached. The consumer's equilibrium was, like the equilibrium of supply and demand, a position of rest, that was "adjusted to" as a result of a relatively dynamic process. This is underscored by the fact that prior to <u>Foundations</u> the second-order conditions for the consumer's constrained optimization problem were generally given as "stability" conditions, while in works written after <u>Foundations</u> that was essentially never the case.
4. "Stability theory" is generally perceived as a failure, rather than a success, for general equilibrium theory.[1] Key counterexamples by Gale (1963) and Scarf (1960)

[1] For the purposes of this paper the term "general equilibrium theory" refers to Walrasian general equilibrium theory of the Arrow-Debreu form, with canonical texts Debreu (1959) and Arrow and Hahn (1971), and summarized recently in McKenzie (2002). In particular, it does not include low-dimensional

demonstrated early on that even well-behaved Walrasian systems could have unstable equilibria. All of the conditions demonstrated to be sufficient for global, and even local, stability were mathematically restrictive and lacked any acceptable interpretation in terms of the underlying characteristics of the economic agents. And finally, the Sonnenschein-Mantel-Debreu (SMD) results on aggregate excess demand functions (Debreu 1974; Mantel 1974, 1977; Sonnenschein 1972, 1973) – that any smooth function satisfying Walras's Law and zero degree homogeneity can be an excess demand function – licensed the production of an unlimited number of general equilibrium models with unstable equilibria.

The paper will argue that these four features of the history of consumer choice theory and general equilibrium theory are not unrelated. In fact – and I believe previously unrecognized – they are all intimately related. More specifically, the paper will argue that #1, #2, and #3 are closely connected, and that has interesting, if somewhat ironic, implications for #4.

The paper is arranged as follows. The first two sections elaborate and support these four characteristics of mid-twentieth century economic theory: the first section focuses primarily on consumer choice theory and the second section discusses the stability of general equilibrium. The third section will offer an explanation of the relationship between these various features, and in particular how they contributed to the stabilzation of consumer choice/demand theory – helped it become <u>the</u> theory, rather than <u>a</u> theory, of consumer choice – in part by deflecting attention away from the issues of path-dependency, endowment effects, non-reversibility, and so forth that were important to economic theorists during the 1930s (and are again in recent years). The story that comes together in this section draws on contemporary history of general equilibrium theory (particularly Weintraub 1991), arguments I have made elsewhere (particularly Hands 1994 and 2006a), as well as pointing out some previously unrecognized connections and linkages. The concluding section will offer some reflections on the overall narrative.

1. <u>Consumer Choice, Demand and Stability</u>

It is useful to start with standard textbook consumer choice thoery and embed it into a Walrasian general equilibrium model, then with that bit of formal scaffolding in place, elaborate on the history and evolution of the relevant theories. In the interest of simplicity I will focus on a pure exchange economy, assume that all prices are strictly positive, all optima are interior, and that all of the relevant functions are differentiable and appropriately well-behaved.

Let the economy have H individuals indexed by $h = 1,2,\ldots,H$ and n commodities $x = (x_1, x_2, \ldots, x_n) \in \Re^n_{++}$ with competitive prices $p = (p_1, p_2, \ldots, p_n) \in \Re^n_{++}$. Each

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Walrasian-inspired Keynesian macroeconomic models of the IS-LM type; representative-agent continuous-equilibrium macroeconomic models of the Lucas variety; any strategic, game theoretic, or imperfect competition-based general equilibrium models; or Walrasian-based models with disequilibrium trading or non-tâtonnement adjustment mechanisms.

individual maximizes a strictly quasi-concave utility function $U^h(x^h)$ subject to a linear budget constraint, so the consumer choice problem is given by,

$$\text{Max } U^h(x^h),$$

$$\text{Subject to } \sum_{i=1}^{n} p_i x_i^h = \sum_{i=1}^{n} p_i \omega_i^h, \tag{1}$$

where $\omega^h = (\omega_1^h, \omega_2^h, \ldots, \omega_n^h) \in \Re_+^n$ is individual $h's$ initial endowment. First-order conditions for the consumer's problem are

$$\frac{\partial U_i^h}{\partial x_i} - \lambda p_i = 0 \text{ for all } i = 1,2,\ldots,n, \tag{2}$$

$$\text{and } \sum_{i=1}^{n} p_i x_i^h = \sum_{i=1}^{n} p_i \omega_i^h.$$

Solving this problem gives us the consumer's demand function for each good,

$$x_i^h = x_i^h(p, I^h) \text{ for } i = 1,2,\ldots,n, \tag{3}$$

where $I^h = \sum_{i=1}^{n} p_i \omega_i^h$ and each x_i^h is homogeneous of degree zero in prices. Under these assumptions the demand functions satisfy the standard Slutsky condition,

$$\frac{\partial x_i^h}{\partial p_j} = S_{ij}^h - z_j^h(p)\frac{\partial x_i^h(p)}{\partial I^h}, \tag{4}$$

where own Slutsky substitution terms are strictly negative ($S_{ii}^h < 0$), the cross-substitution terms are symmetric ($S_{ij} = S_{ji}$ for all $i \neq j$), and the $n \times n$ matrix of Slutsky substitution terms $S^h = \left[S_{ij}^h\right]$, is negative semi-definite ($x^T S^h x \leq 0$ for all $x \neq 0$).

The optimal solution to the consumer choice problem is still called the consumer's equilibrium in a few texts, but these days it is far more likely to be called the consumer's "optimal choice" or simply "solution." During the 1920s and 1930s it was standard to use the term "equilibrium" for the consumer's optimal position within the choice space. If one views an "equilibrium" as a position of rest of a dynamic system, then it is not obvious in what sense the consumer's optimal solution is an equilibrium. If I want (prefer) to pick up the pencil on my desk, and do so, how am I in equilibrium? The stability literature of the 1950s and 1960s characterized general equilibrium as a rest point of a formal dynamic process involving time and differential or difference equations, and consumer choice theory certainly contains no dynamics in this formal sense. Yet, as we will see, early in the twentieth century the consumer choice problem was often couched in terms of movement and change in a way that gave it significant "dynamic" element (a dynamic element that later disappeared from consumer choice).

Vilfredo Pareto's discussion of economic equilibrium in the *Manual* (1971) provides a nice example of such a movement-based characterization economic choice. In chapter three he described economic equilibrium in the following way.

> Given the economic state, if we were able to *move* away from it by any kind of *movement* whatsoever, we could continue indefinitely *movements* which increase the quantities of all goods which man may desire; in this way we would *reach* a state in which man would have everything to satiety. Obviously this would be a position of equilibrium; but it is also obvious that things do happen that way in real life, and that we will have to determine other positions of equilibrium at which *one* *must* *stop*, because only certain *movements* are possible. In other words, there are obstacles which limit *movements*, which do not allow man to follow certain *paths*, which prevent certain *variations* from taking place. Equilibrium results precisely from this opposition of tastes and obstacles. (Pareto, 1971, p. 109, emphasis added)

And similarly on the next page:

> To determine equilibrium we will set up the condition that at the moment when it occurs, *movements* permitted by the obstacles are prevented by the tastes; or conversely, what comes to the same thing, that at this moment, *movements* permitted by the tastes are prevented by the obstacles. Indeed, it is obvious that in these two ways we express the condition that *no* *movement* *occurs*, and this is, by definition, the equilibrium characteristic. (ibid., p. 110, emphasis added)

Notice that the economic agent is characterized as moving about in the choice space directed by tastes and limited by obstacles. Given a common sense definition of the word "dynamic" the agent is involved in a dynamic process of moving toward equilibrium and equilibrium is a position from which no further movement takes place. There are no explicit dynamics here – the agent's behavior is not specified explicitly in terms of differential or difference equations – and yet it is, in a general sense, quite dynamic.

R. G. D. Allen provides another clear example from this period. Allen's methodological approach made him particularly insistent about the dynamic nature of inter-agent equilibrium.

> An individual economic action can be expressed quantitatively as a *change* from one combination, in which the individual possesses amounts $x, y, z, \_\_$ of the m goods, to another combination in which the individual possesses amounts $x+\Delta x, y+\Delta y, z+\Delta z \_\_$ of the goods. … The result is an economic *action*, in fact, may be that the amount possessed of any one

> good has increased, decreased, or remained constant. The expression of an economic action corresponds to a "movement" from the point $P(x, y, z\_\_)$ to the point $Q(x + \Delta x, y + \Delta y, z + \Delta z, \_\_)$. (Allen, 1932, p. 200, emphasis added)

This dynamic characterization is particularly important for Allen since he insisted – following the lead of physics – on starting from finite, observable, movements and then "going to the limit" to convert these finite changes into differential equations which could then be "integrated back" to recover the underlying forces that generated the movement.

> In all sciences, observed or experimental facts are expressed directly in terms of finite differences and finite difference equations, but before the mathematical theory can be constructed these must be translated into differentials and differential equations, which are alone susceptible to mathematical analysis in its more powerful forms. (ibid., p. 201)
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> In mechanics, observed phenomena consist of movements of particles and bodies, and the object of the mathematical theory is to explain these movements by the invention of hypothetical causes and causal relations. … In economics, observed phenomena consist of economic movements or exchanges, and pure economic theory must explain these movements by inventing hypothetical causes. Starting from the vague idea of individual wants and desires as the cause of economic movements, we are led to the mathematical definition of the individual's preference direction outlined above. By integration, … individual total utility is derived as the "potential" of economic theory. (ibid., p. 208, emphasis added)

For Allen movement was thus an essential and ineliminable aspect of the scientific theory of consumer choice. If the consumer is not moving in the choice space there are no differential equations to work with and thus no science; observed systematic movement is, on this view, a precondition for the application of mathematics to scientific phenomena. Although Allen is probably exceptional in the degree of his commitment to the physics metaphor (Mirowski 1989), the point is that the characterization of consumer choice as fundamentally a dynamic process of moving toward – and if displaced, returning to – and equilibrium position was quite common in the literature of the 1920s and 1930s.

The change becomes even more clear when we consider second-order conditions for the consumer's optimization problem. The assumption that the utility function is strictly quasi-concave is now typically called a second-order condition – it is a condition, that when combined with the first-order conditions, is jointly sufficient to guarantee that the critical point given by (2) is in fact a solution to the problem (1) – it is equivalent to all indifference surfaces being convex to the origin (and a number of equivalent mathematical conditions). Early in the twentieth century when the details of this, now standard, characterization of consumer choice were being worked out, it was customary to call this second-order, or concavity, condition a stability condition. The idea being that if the second-order condition is satisfied, any displacement away from the optimal bundle is automatically pulled back toward the optimum. As Eugene Slutsky explained:

> It is clear, besides, that the one possible state in which an individual's budget could remain unchanged, even for a short time, is that whose present utility is equal to or greater than that of all states immediately proximate. Such a state can be called a state of equilibrium. It is <u>stable</u> if any divergence from it tends to diminish utility, <u>unstable</u> in the contrary case. (Slutsky, 1915, p. 29)

Similarly from Jacob Mosak:

> In order that the equilibrium position should be <u>stable</u>, it is necessary that the point represent a most preferred rather than least preferred combination of goods. This means that any <u>departure</u> from the equilibrium point must lead to a less preferred position. … Geometrically this implies that the indifference surfaces must be convex to the origin in the neighborhood of equilibrium … In order that U shall be a true maximum rather than a minimum or minimax we must also have the <u>second-order or stability conditions</u>. (Mosak, 1944, pp. 11-12, emphasis added)

During the early twentieth century equating second-order and stability conditions was standard in the literature, including the works that eventually became the core documents in budget-constrained utility-maximizing model of consumer choice (Allen 1932, 1938; Hicks 1946; Hicks and Allen 1934; Mosak 1944; Schultz 1938; and Slutsky 1915).

As Roy Weintraub (1991) has explained in detail, the notion of "equilibrium" in economics – and the associated concept of "stability" – did not emerge fully developed at some particular moment early in the twentieth century. There were multiple conceptions of equilibrium and stability vying for professional attention (often in the work of a single author or text), and as a result the use of these terms – from a contemporary, post-stabilization, perspective – seems to be ambiguous and fraught with various tensions.[2] In particular, two distinct images of equilibrium that were often blended in various ways were "the imagery of agents acting to make themselves better off with the image of a state of rest" (Weintraub, 1991, p. 101) – the "distinction between equilibrium as a behavioral outcome and equilibrium as a mechanical rest point" (ibid., p. 103). Equating the second-order/concavity conditions with the stability of the consumer's equilibrium is a clear example of intertwining these two images.

But there was yet another notion of stability in early consumer choice theory – the "stability" of the underlying preferences – and here too various tensions and ambiguities were also at work. If one assumes the consumer "has" an ordinal utility function, or

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[2] And in this paper the issues are only consumer choice theory and the stability of general equilibrium (discussed below). If one includes the various concepts of equilibrium and stability percolating about in capital theory, long run growth theory, and macroeconomics (business cycle theory), the tensions and ambiguities become even more pronounced.

complete and transitive preferences (that can thus be represented by such a function), then the consumer exhibits a degree of "stability" as they move about in the choice space and respond to various changes in parameters (prices). If the consumer chooses bundle A then moves to B, but then goes back to A, the level of utility (or more to be more ordinal: whether they prefer A to B, B to A, or are indifferent) will remain the same. That is what a <u>function</u> means: same independent variable, same dependent variable. In this sense "stability" means essentially that preferences (and the associated utility function) do not change with changes in the endowment, commodity bundle, the process of decision making, or what contemporary behavioral economists call the "elicitation process." Such stability was implicit in the above (and any standard) presentation of the consumer choice problem. Starting with $U^h(x^h)$, and retaining it throughout, is implicitly a stability assumption on the preferences of the consumer.

This gives us at least three notions of stability at work within the early theory of consumer choice: The optimal consumption bundle as a state of rest (a position from which there would be no tendency to change); the mathematical properties, the concavity/second-order conditions, on the utility function that are sufficient for the critical point to be a maximum; and the stability of the underlying preferences/utility function (over all bundles in the choice space, and during the time-frame of the choice problem). In the literature of the early twentieth-century these three conditions were all interacting in various ways in a broadly dynamic characterization of consumer choice.

For someone like Allen the stability of preferences independent of the endowment and consumption bundle presented a serious epistemological problem. What one started with for Allen – the <u>empirical</u> facts of the matter – were the observable price ratios, the marginal rate of substitution, at the various consumption bundles. But these observations, even if they exhibited a clear pattern, were simply – to use Paul Samuelson's (1950) term – little local "thumbtacks," and not a complete picture of the consumer's indifference surfaces (or $U^h(x^h)$) over the entire choice space. By going to the limit one could derive from these finite observations a differential equation that would express the change in the consumer's utility index for infinitesimal changes in the consumption bundle, but what one could not do – at least without additional assumptions – was obtain a complete picture of the (global) indifference surfaces and/or the underlying utility/potential function. For this one needed to know that the relevant equation was <u>integrable</u>, and that information was not available from only the observables of the problem. As Allen explains

> The equation I express the relation between the components of an infinitesimal movement, $dx, dy, dz, \_\_$, about which the individual is indifferent. The equation need not be integrable, so that it is impossible, in general, to pass from infinitesimal "indifferent" movements at a particular point to the indifference <u>loci</u> in bulk. The indifference <u>loci</u> can only be given, in general, in infinitesimal portions by means of the differential equation I. If the differential equation is to be integrable, it is necessary to make an additional assumption to this effect. (Allen, 1932, p. 222)

In case the reader suspects that such issues only concerned Allen during the early, pre-Hicks and Allen (1934), days, and were later dropped, the following quote from the 1950 printing of Allen's 1938 mathematical economics textbook demonstrates his consistency on this matter (also see Fernandez-Grela 2006).

> In general, therefore, we cannot integrate the set of indifference planes into a complete set of indifference surfaces, and we cannot assume that any utility function exists. The assumption of a scale of preferences for small changes of purchases does <u>not</u> imply that a complete scale of preferences exists. The consumer can discriminate between small changes from his established purchases but need not be able to discriminate between widely different sets of purchases. (Allen, 1950, pp. 440-41)

This is one version of the <u>integrability</u> <u>problem</u> – the version that I have elsewhere called the "thumbtack integrability problem" (Hands 2006a) – and for Allen and many others it was a serious challenge to budget-constrained utility-maximization based consumer choice and associated demand theory. One solution of course – and the solution that probably strikes contemporary economists as most obvious – is to simply <u>assume</u> that the consumers have well-ordered (complete and transitive) preferences which can then be represented by a well-behaved ordinal utility function (Debreu 1954). This still leaves open one version of the "stability" problem – the problem of the second-order/concavity conditions – since completeness and transitivity only guarantee the existence, not the strict quasi-concavity, of the utility function, but it does solve (actually dissolve) the thumbtack integrability problem. If the consumer has well-ordered preferences the thumbtack integrability problem simply goes away. Obviously for someone with Allen's particular empiricist commitments, this is not an acceptable solution since such an assumption is just that, an assumption, and not a fact that is (or even could be) available from the empirical evidence (which is always at best local and finite). But even many economists less epistemically persnickety than Allen were also quite skeptical about simply starting with the assumption that the consumer has well-ordered preferences defined over the entire choice space.

The integrability problem for many economists working on consumer choice early in the twentieth century was not simply the thumbtack integrability problem discussed above, but rather it was a much broader class of problems – one that I previously labeled integrability$_B$ for the "broad" integrability problem (Hands 2006a) – the problems that we now associate with the various anomalies that have repeatedly been identified in experimental psychology, experimental economics, and behavioral economics: context-effects, sensory thresholds, reference-dependency, endowment effects, loss aversion, framing, non-reversibility, and such. In a nutshell these are all issues that have recently received attention because agents in laboratory environments consistently fail to behave in the way one would expect if they were maximizing a well-behaved stable utility function. Translating such anomalies into Allen's language from above, the differential equation that describes their choices must not form an integrable preference field – if it did, the choices could be rationalized by constrained maximization of well-behaved utility function and such "anomalies" or "violations" would not occur.

Although the contemporary literature on experimental and behavioral economics is consistently couched in terms of risky choice and expected utility theory, and based on laboratory experiments that were not available in the 1930s,[3] the core issues are much the same as those associated with integrability$_B$ and they represent a similar challenge the adequacy of the standard stable well-ordered preference-based framework for explaining individual behavior. As Daniel Kahneman explained in his Nobel lecture.

> The proposition that decision makers evaluate outcomes by the utility of final asset positions has been retained in economic analysis for almost 300 years. This is rather remarkable, because the idea is easily shown to be wrong; I call it Bernoulli's error. (Kahneman, 2003, p. 1455)
>
> Bernoulli's error – the idea that the carriers of utility are final states – is not restricted to decision making under risk. … The error of reference-independence is built into the standard representation of indifference maps. It is puzzling to a psychologist that these maps do not include a representation of the decision maker's current holdings of various goods – the counterpart of the reference point in prospect theory. The parameter is not included, of course, because consumer theory assumes that it does not matter. (ibid., p. 1457)

These same motivations – developing a choice theory that avoided the integrability$_B$ issues of (in)stability, (lack of) symmetry, and (ir)revesibility – motivated much of the work on consumer choice theory in the 1920s and 1930s: Allen's (1932, 1936, 1938) insistence that the integrable case was only a "special" case, Griffith Evan's (1930) non-utility based approach to demand, Ragnar Frisch's (1971: originally published in 1926) model of contingent choice, Nicholas Georgescu-Roegen's (1936) psychological threshold-based argument for directed choice, and even Vilfredo Pareto's (1971: translated from the 1927 edition) discussion of the order of consumption, are all examples of these concerns (and this is not an exhaustive list). It is not necessary to go into the details of these models (see Hands 2006a for details) to recognize that they represented the same broad class of issues that concern many contemporary behavioral economists. Of course none of these earlier approaches ever won the hearts and minds of the profession, as evidenced by the fact that discussion of these problems and the associated effort to reformulate choice theory in a way that avoided such difficulties virtually stopped (at least within the mainstream literature) until the experimentally-inspired revival of the last few decades.

Although it is seldom recognized that many of the early twentieth-century contributors to consumer choice theory were concerned about integrability issues, or that many of the broad integrability issues they emphasized were essentially the same as those that have re-emerged within the recent experimental and behavioral literature, it is generally recognized that these issues were almost never discussed during the 1950s and 1960s.

[3] Although there were a few early experiments. See Moscati (2007) for a detailed discussion.

The heyday of Arrow-Debreu general equilibrium theory was also the heyday of stable well-ordered preferences.

Another fact that is seldom noticed is point #3 from the introductory section above – that after Foundations second-order conditions were no longer called stability conditions. In chapter 5 of Foundations – the pure theory of consumer behavior – Samuelson does refer to the first order conditions for the budget-constrained maximization of the consumer's utility function, given in (2) above, as "equilibrium" conditions, but the bordered Hessian restrictions that guarantee the vector satisfying (2) is in fact a maximum, are nowhere called a "stability" conditions. The relevant necessary and sufficient conditions for a constrained n-variable maximization problem are called just that: necessary and sufficient conditions. Part I of Foundations examined optimization-based models (without any discussion of dynamics or stability), while part II was dedicated entirely to the stability properties of dynamic, non-optimization-based, models (models examined in the next section). Maximization and minimization are one thing – one way of generating "meaningful" comparative statics results – and stable dynamic models are another thing entirely (but can also provide meaningful comparative statics results). It is important to be clear that my argument is not that these changes were exclusively Samuelson's. For example, Oscar Lange's (1944) Price Flexibility and Employment is a very influential text that does things in the Foundations way – strict separation of static maximization and dynamic stability – and yet it appeared in print before Foundations. This is not a story about the impact of one individual economist, it is about a sea change in economic theory involving the stabilization and standardization of the concepts of equilibrium and stability. One of the main tasks of this paper is to fill in the hows, whys, an unintended consequences of these changes.

The influential advanced textbooks of the immediate post-war period followed Samuelson's lead. Bushaw and Clower (1957), Henderson and Quandt (1958) and Chiang (1984, first edition 1967) all referred to second-order conditions for optimization problems as second-order conditions (not stability conditions) and reserved the term stability for dynamic systems involving differential (or difference) equations. While carry-over textbooks such as Allen (1938) continued to use the term "stability conditions" for second-order conditions and characterize consumer choice in a dynamic way, that was not the case for the newer literature. Books written after World War II, after the acceptance of the neoclassical synthesis, generally kept a strict separation between optimization and stability. If one moves farther forward in time to Takayama (1974) or recent advanced textbooks such as Mas-Colell, Whinston, and Green (1995) the separation becomes even more pronounced; now not only has "stability" disappeared from consumer choice theory, but the word "equilibrium" has disappeared as well.

2. General Equilibrium, True Dynamic Stability, and Consumer Choice

The consumer choice problem and the corresponding demand functions form the cornerstones of the Arrow-Debreu general equilibrium model. From (3) we have the excess demand for good i by individual h given by,

$$z_i^h(p) = x_i^h(p) - \omega_i^h, \tag{5}$$

where $x_i^h(p)$ is the demand for, and ω_i^h the initial endowment of, good i by individual h. Summing over the H individual the market excess demand function for good i is given by,

$$z_i(p) = \sum_{h=1}^{H} z_i^h(p).$$

The main properties of market excess demand functions are zero degree homogeneity (H) and Walras's Law (W) for all prices:

$$z_i(p) = z_i(\lambda p) \text{ for all } \lambda > 0 \text{ and for all } i = 1,2,\ldots,n, \tag{H}$$

$$p^T z(p) = \sum_{i=1}^{n} p_i z_i(p) = 0. \tag{W}$$

In the pure exchange case both of these conditions follow immediately from the structure of the underlying consumer choice problems.

The homogeneity assumption allows us to normalize prices by defining all prices in terms of good n, making $p_n = 1$. If, for simplicity, we continue to use the same symbol (p) for normalized prices, the price vector becomes $p = (p_i, p_2, \ldots, p_{n-1}, 1)$. Hereafter we will assume that all prices are normalized prices.

After this normalization the $(n-1)\times(n-1)$ excess demand Jacobian $Jz(p)$ is nonsingular, so

$$|Jz(p)| \neq 0 \quad \text{with} \quad Jz(p) = \begin{bmatrix} z_{11} & \cdots & z_{1n} \\ \vdots & & \vdots \\ z_{n1} & \cdots & z_{nn} \end{bmatrix}, \tag{6}$$

$$\text{and} \quad z_{ij} = \frac{\partial z_i(p)}{\partial p_j} \text{ for all } i, j = 1,2,\ldots,n-1.$$

In general competitive equilibrium we have:

$$z_i(p^*) = 0 \text{ for all } i = 1,2,\ldots,n-1, \tag{7}$$

where $p^* = (p_1^*, p_2^*, \ldots, p_{n-1}^*, 1)$ is the equilibrium price vector. The well-known existence theorem of Arrow and Debreu (1954) guarantees that an equilibrium price vector always exists under these standard assumptions.

The question of how competitive markets reach equilibrium goes back to at least the 18th century, and a variety of explanations/adjustment mechanisms have been proposed during the long history of the subject (price adjustment, quantity adjustment, single market, multiple market, long run equilibrium, short run equilibrium, with disequilibrium trading, without disequilibrium trading, with recontracting, without recontracting, equilibrium in stocks, equilibrium in flows, …). By the early 1950s economic theorists had reached a fairly general agreement about the proper framework for analyzing one particular version of the problem: the price adjustment mechanism for a Walrasian multiple market competitive equilibrium model. The framework was the Walrasian tâtonnement introduced by Paul Samuelson in his paper on "true dynamic stability" in 1941. Samuelson further elaborated the model in 1942, 1944 and 1947, and it was quickly adopted by Arrow and Hurwicz (1958), Lange (1944), Metzler (1945) and became the standard approach in the literature. The Walrasian tâtonnement is given by the following system of n first order ordinary differential equations (t=time):

$$\frac{dp_i(t)}{dt} = z_i[p(t)] \text{ for all } p \text{ and for all } i.^4 \qquad \text{(T)}$$

This system of differential equations captures the idea that the price of a particular good should increase when it has positive excess demand, decrease when it has negative excess demand, and be in equilibrium when excess demand is equal to zero. The economic interpretation of (T) is that it represents a type of auctioneer's rule, where the auctioneer (or "the market") adjusts prices on the basis of the relationship between supply and demand until the equilibrium p^* is reached (where the process stops). The model is competitive in that both suppliers and demanders take prices as parameters and there is no disequilibrium trading; the tâtonnement process changes prices until the equilibrium is reached, then exchange takes place.

The solution to this system of differential equations is given by,

$$p(t) = (p_1(t), p_2(t), \ldots, p_n(t)).$$

The problem of stability – asymptotic stability – is the question of whether the solution converges to the equilibrium price vector over time. In other words, whether it is the case that

$$\lim_{t\to\infty} p(t) = p^*. \qquad (8)$$

If (8) holds for any initial price vector $p(0) \neq p^*$ starting anywhere in the domain, then the equilibrium is globally stable. If it is only that there exists $\varepsilon > 0$ such that (8) holds for any initial price vector $p(0) \neq p^*$ within this ε-neighborhood of p^*, then the

[4] This is the simplest of many tâtonnement mechanisms used in the literature. Others include $\frac{dp_i}{dt} = k_i z_i[p(t)]$ with $k_i > 0$, and $\frac{dp_i}{dt} = H_i[z_i[p(t)]]$ where $signH_i = signz_i$.

equilibrium is locally stable.[5] The main results on the global stability of general equilibrium are based on Liapunov's second method, introduced into the Walrasian literature by Arrow and Hurwicz (1958).[6] The main mathematical condition for local stability of (8) is that the characteristics roots of Jacobian matrix associated with the dynamical system must have negative real parts at the equilibrium. In other words

$$\text{if } \operatorname{Re}\lambda_i(JZ) < 0 \text{ for all i,} \tag{9}$$

where the matrix JZ is the Jacobian in (6) evaluated at the equilibrium price vector p^*.[7] There are a number of mathematical restrictions on JZ that imply (9), a number of economically interpretable conditions on excess demand functions that imply these mathematical restrictions, and in turn a number of restrictions on the preferences of the agents that will generate excess demand functions that have the desirable properties (some of these are discussed below).

In his 1941 paper Samuelson contrasted the "true dynamic" stability of the adjustment mechanism (T) with the "perfect stability" that John Hicks introduced in Value and Capital (1946, first edition 1939). Hicks was the first to provide a stability condition for an n-good Walrasian general equilibrium model, but his condition was not based on the "true dynamic" adjustment mechanism (T) or the associated stability condition (9). As Weintraub explains "For Hicks, stability was process-independent; the formal analysis (as opposed to the verbal exposition) was entirely concerned with the character of equilibrium positions" (Weintraub, 1991, p. 35).

Hicks's condition was also a restriction on the excess demand Jacobian at equilibrium (JZ) – the condition that all principal minors of JZ alternate in sign starting negative – but that condition is not, in general, equivalent to (9).[8] Such matrices have gone by many different names in the various literatures where they have appeared – including being called "Hicksian" – but the most common label is "NP-matrix." Since we will need to

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[5] Again, these are only the most important stability concepts. Many others were also employed in the general equilibrium literature: stability independent of the adjustment speed (k_i), stability of the process (some equilibrium would always be reached although there may be more than one), orbital stability (the price path $p(t)$ forms a closed curve, but p^* is not on that curve), structural stability (the qualitative properties of the model – number of equilibria and such – remain invariant for variation in the parameters), and others. It is also possible that local stability is sensitive to the choice of numeraire (Mukherji 1973). Finally, it should be noted that the adjustment mechanism can be expressed in terms of difference, rather than differential, equations.

[6] See Weintraub (1991) for a history of Liapunov's method and its application in economics.

[7] This condition is both necessary and sufficient for the local stability of the linear approximation of the system (T) – the system $dp/dt = JZ(p(t) - p^*)$ based on taking the Taylor series expansion of the right hand side of (T) around the equilibrium p^* -- but it is only sufficient for the local stability of the underlying (non-linearly approximated) system (T). Sufficiency is usually adequate since much of the local stability literature in general equilibrium theory has amounted to finding economically interpretable condition that imply (9).

[8] It was demonstrated that (9) did not imply the Hicksian condition in Samuelson (1941) and that the Hicksian condition did not imply (9) in Samuelson (1944). The two conditions are equivalent if the matrix is symmetric – if $Z_{ij} = Z_{ji}$ for all $i \neq j$.

refer to this class of matrices at several points in the discussion below it is useful to write out the Hicksian perfect stability condition:

$$JZ \text{ in an NP-matrix, that is } signM_i(JZ) = (-1)^i \text{ for all i,} \qquad \text{(NP)}$$

where $M_i(JZ)$ is an ith order principal minor of the matrix JZ.

It is clear that early on in the investigation of the stability of general equilibrium most of those working in the field hoped to prove that the Walrasian tâtonnement would be locally, and perhaps even globally, stable under fairly weak restrictions on excess demand functions and/or the preferences of the underlying agents. Ideally, it was hoped that simply assuming all individuals had well-ordered preferences and solved the standard consumer choice problem (1) would do the job. Since Arrow and Debreu had proven that the equilibrium price vector $p*$ always exists, and the first fundamental theorem of welfare economics guaranteed that $p*$ would be associated with a Pareto Optimal allocation of resources, the ideal result would be that the combination of rational agents, competitive prices, and the law of supply and demand (T), would be sufficient to guarantee stability as well. But low, it was not to be. Local, and even global, stability was demonstrated for a number of special cases, but these results were never very satisfying. These conditions include:

- Gross Substitutes (GS):

$$\text{if } Z_{ij} = \frac{\partial z_i}{\partial p_j} > 0 \text{ for all } i \neq j.$$

- Weak Axiom of Revealed Preference (WARP):[9]

$$p* z(p) = \sum_{i=1}^{n} p_i^* z_i(p) > 0 \text{ for all } p.$$

- Symmetric Hicksian Case (SH):

$$\text{If } Jz(p) \text{ is symmetric, } Z_{ij} = Z_{ji} \text{ for all } i \neq j \text{ and NP.}$$

- Dominant Diagonal (DD)

$$|Z_{ii}| > \sum_{\substack{j=1 \\ j \neq i}}^{n} |Z_{ij}| \text{ for all } i = 1,2,\ldots,n.$$

In most cases, if the condition holds for all prices it implies global stability and if it hold only at $p*$ it implies local stability.[10]

[9] Note, the weak axiom of revealed preference holding for individual consumers is not a particularly restrictive assumption, but WARP imposes the weak axiom on aggregate excess demand functions and that is a restrictive assumption. When the stronger version of WARP – the strong axiom (SARP) – is imposed on aggregate excess demand functions it essentially means that the economy behaves as if there is just one big consumer.

[10] I will make no attempt to cite the many contributions that were made to the literature on the stability of general equilibrium. A number of surveys are available within the theoretical literature – Arrow and Hahn

The problem is that none of these conditions, or any of the many others available in the literature provided the kind of simple, economically acceptable, restriction that general equilibrium theorists hoped would be sufficient for stability. The problem was exacerbated by two influential papers – Gale (1963) and Scarf (1960) – which provided fairly simple general equilibrium models which satisfied the standard assumptions such as (H) and (W), and yet had locally unstable equilibria. By 1971 Kenneth Arrow and Frank Hahn would end their two chapter-long discussion of the stability of competitive equilibrium with the following rather downbeat assessment

> Although we set ourselves the task of investigating "the price mechanism" in a highly simplified setting, it will probably be agreed that the task is not simple, and that it has not been definitively completed. There is a distressingly anecdotal air about our investigation; case succeeds case, but it was not found possible to lay down any general principles. (Arrow and Hahn, 1971, p. 321)

A few years after Arrow and Hahn's General Competitive Analysis a series papers appeared that effectively dealt the death blow to stability analysis in general equilibrium theory (at least any analysis based on the Walrasian tâtonnement and employing aggregate excess demand functions). The papers are generally called the Sonnenschein-Mantel-Debreu (or SMD) results on aggregate excess demand functions after the authors of the most important papers in the literature: Sonnenschein (1972, 1973), Mantel (1974, 1977), and Debreu (1974).[11] Although each contribution to the SMD literature arrived at slightly different results for slightly different models, the general theme is that almost any function can be an excess demand function. In other words, the SMD results say that any smooth function that is homogeneous of degree zero (H) and satisfies Walras's law (W) can be an excess demand function (in the sense that it could have been generated by the constrained utility maximization by well-behaved neoclassical agents). This opens the door to the construction of an infinite number of counterexamples to stability; find any mathematically well-behaved function that satisfies (H) and (W) that does not satisfy (9) and you have a counterexample. It is easy to see why the SMD results became "a sort of leitmotiv (or nightmare) running through all research into … stability" (Ingrao and Israel, 1990, p. 317). The bottom line is that the SMD results have been quite negative for stability theory – of course things were fairly bleak even before the SMD literature – and without stability there is no reason to believe that equilibrium price vector would be reached, or another equilibrium found after even a small change in parameters, and this does not bode well for the entire general equilibrium program. As Alan Kirman recently put it:

(1971. Chs. 11 and 12), Hahn (1982), Negishi (1962), Newman (1959, 1962), Varian (1982) – and Ingao and Israel (1990, Ch. 12) and Weitraub (1991) provide detailed historical discussions. See Ikeo (1994) for a discussion of the Japanese stability literature.

[11] See Shafer and Sonnenschein (1982) for a survey. Kirman (1989, 2006) and Rizvi (2006) provide discussions of the broader implications for economic theory.

> The full force of the Sonnenschein, Mantel, and Debreu (SMD) result is often not appreciated. Without stability or uniqueness, the intrinsic interest of economic analysis based on the general equilibrium model is extremely limited. (Kirman, 2006, p. 257)

3. Stabilizing Consumer Choice

The previous two sections have provided some elaboration and documentation for the various pieces of the puzzle; it is now time to put the puzzle together. The story goes like this.

During the 1920s and 1930s the constrained utility maximization-based theory of consumer choice was more dynamic than in the post-*Foundations* literature. This is certainly not to suggest that there was a single dynamic characterization that was accepted by the majority of economic theorists. There was no such consensus on *any* particular characterization or formalism; this was a time of pluralism and theoretical diversity before the stabilization that took place after World War II (Mirowski and Hands 2006, Morgan and Rutherford 1998).[12] The claim is simply that all across this theoretical diversity the consumer was consistently characterized in a far more dynamic way than was the case in say, 1960, or in textbooks today. Of course there were no formal dynamics (dynamics that would be recognized as such after the tâtonnement literature), but the consumer was generally viewed as moving through the choice space in the direction of the optimal bundle and once it was reached kept in place by stabilizing forces. This dynamic characterization of the economic agent brought time into consumer choice and opened the whole framework to various criticisms associated with position, context, reference dependency, order of consumption, threshold effects, and such: the problems of integrability$_B$. These issues have of course recently returned to the theory of economic behavior – now driven by evidence garnered from agents in experimental laboratories – but it has been a long hiatus.

Samuelson's 1941 paper provided a very specific way of conceptualizing stability in economic models that was very influential and over time contributed to economists thinking about dynamic "stability" exclusively in terms of differential (or difference) equations and about the "equilibrium" in such models as the rest point of a formal dynamic system. This basic framework had its most obvious impact on the analysis of the stability of the Walrasian tâtonnement – as witnessed by the series of papers from Metzler (1945) to Arrow and Hurwicz (1958), to Arrow, Block, and Hurwicz (1959) all recognizing and explicitly trying to build upon Samuelson's dynamic framework – but it eventually came to influence "dynamic" modeling in every area of formalized economic theory. This was certainly not Samuelson's doing alone. As Weintraub (1991) explains

[12] Of course some of the pluralism came from Marxists, Institutionalists, and other alternative economic research programs, but that is not what is meant by "pluralism and theoretical diversity" here. The period from the turn of the century to World War II was a period of theoretical diversity *even among those who generally considered themselves neoclassical* (or as they would have said at the time "marginalists") and were drawn toward mathematical formalization.

in detail, the transformation from the buzzing confusion over equilibrium and stability in the early twentieth century, to the stabilized dynamics of the post-war period involved many individuals, over a long period of time, involving a variety of intellectual and social forces, and has no single source. Of course wars and revolutions are products of extremely complex forces involving many individuals over a long period of time as well, and yet they have flash points and declarations. Samuelson's characterization of stability and dynamics was such a flash point; not so much the original papers – Samuelson 1941, 1942, 1944 – but certainly with the publication of Foundations in 1947 (these earlier stability papers were reprinted with only a few minor changes in chapter 9 and 10 of Foundations).

By the 1950s "dynamic" meant "based explicitly on differential or difference equations involving time," and optimization problems – maximum or minimum – were not of this sort. Maximization was not a dynamic process; the Walrasian tâtonnement was. The entire structure of Foundations lent itself to the strict separation of optimization and dynamics; as noted above, Part I was explicitly about optimization-based problems and Part II explicitly about dynamics. There was a unified mathematical framework for deriving qualitative comparative statics results from both optimization-based and dynamic models, but the mathematical structure required to obtain such results differed in the two cases. In the case of optimization problems like consumer choice theory, the necessary structure came from the problem's second-order conditions (the "maximization hypothesis"); in the case of dynamic models the necessary structure came from stability conditions (what Samuelson called the "Correspondence Principle"). There was a common mathematical framework that subsumed both classes of models, but the two classes of models were quite different.

The ultimate impact of this separation – or the impact of the profession generally accepting this separation[13] – was that consumer choice theory, which was based on utility maximization, ceased to have anything to do with movement or dynamics. Of course no dynamics means no paths, no endowment effects, no reference dependence, no order of consumption, none of the other problems associated with integrability$_B$. The concept of economic dynamics is stabilized and in the process consumer choice theory is relieved of the responsibility for dealing with all of these potentially troublesome issues. Economic agents with well-ordered preferences defined over the entire choice space became the standard basis for consumer choice theory and the non-integrable case and all the difficulties associated with it quietly left the stage. Stabilizing dynamics thus helped stabilize consumer choice.

While Samuelson's Foundations and the literature that followed did separate optimization and dynamics, there are a few places – in Foundations and elsewhere in the

[13] It is interesting to emphasize that the separation was maintained even though the main technical results in the two areas were produced by essentially the same people, during the same period of time, and published in the same journals. Arrow and Entoven (1961) provided the formal characterization second-order conditions in terms of the quasi-concavity of the objective function in Econometrica at roughly the same time that Arrow's work on the global stability of the Walrasian tâtonnement wad published in the same journal.

literature – where the two touched briefly, and these points of contact turn out to be particularly informative regarding the concerns of motivations of the theorists involved. The most obvious example of such contact involves a particular form of the dynamic system (T) – a "gradient system" where the Jacobian matrix is symmetric. Recall the Walrasian tâtonnement:

$$\frac{dp_i(t)}{dt} = z_i[p(t)] \text{ for all } p \text{ and for all } i = 1,2,\ldots,n. \tag{T}$$

A special case of such a dynamical system is the case where the Jacobian matrix is symmetric:

$$Z_{ij} = Z_{ji} \text{ for all } i \neq j \text{ where } z_{ij} = \frac{\partial z_i(p)}{\partial p_j}. \tag{S}$$

Note that (S) generally does not hold on the excess demand functions of Walrasian models (even in the pure exchange). Changes in the prices of goods have both substitution and income effects; the substitution effects, given by (4) above, are symmetric, but the income effects in general are not (nor do they, in general, vanish). (S) holds in some special cases – such as homothetic preferences – but certainly not in general. The condition (S) might be imposed globally – on $Jz(p)$ throughout the price domain – or only locally (on $JZ = Jz(p^*)$).

The symmetry condition (S) implies that a potential function $F(x)$ exists where each of the excess demand functions is a first derivative of the potential function F. Thus,

$$\text{(S) implies } \exists F(x) \text{ such that } z_i(p) = \frac{\partial F(p)}{\partial p_i} \text{ for all } p. \tag{10}$$

But this means that,

$$z_i(p^*) = \frac{\partial F(p^*)}{\partial p_i} = 0 \text{ for all } i = 1,2,\ldots,n,$$

and thus, in this particular case, the equilibrium price vector p^* – a rest point for the dynamic system (T) – is also a critical point for the potential function F. This is clearly a model where there is no strict separation between optimization and equilibrium. The critical point (first-order condition) for the optimization of F is the equilibrium (rest point) of the dynamic system (T). In fact we can go a bit farther; this is a case where the local stability of the dynamic system (T) is equivalent to the maximization of some function (in this case F).

Standard results from optimization theory tell us that the critical point is a maximum if the associated Hessian matrix $HF(p^*)$ is negative definite, a condition equivalent to having principal minors alternate in sign starting negatively (i.e. it is an NP-matrix). But recall the stability condition from (9) above. Under the symmetry condition (S) the requirement of having the real parts of all characteristic roots negative is equivalent to JZ being negative definite, which is in turn equivalent to the Hicksian stability condition (NP). But given (10) the Hessian of the optimization problem is the Jacobian of the dynamic problem – $HF(p^*) = JZ$ – and thus, in this case, stability implies maximization and maximization implies stability. For such symmetric, or gradient, systems there is a formal equivalence between maximization and the stability of the true dynamic system.[14]

Samuelson does discuss this result – in Samuelson (1941) and pp. 270-72 and 301-02 of Foundations – but it is only mentioned in passing and as a special case. Weintraub examines it in detail (1991, pp. 54-66) since it also represents the place where Samuelson came very close to employing the Liapunov technique that was used later in the analysis of global stability. Weintraub gives two main reasons why Samuelson did not do more with the symmetric case: the fact that he was primarily interested in comparative statics and the influence of E. B. Wilson. These do seem to be important factors, but in Hands (1994) I added two more: 1) Samuelson wanted to provide foundations for Keynesian (and for that matter Walrasian) systems which generally lack the symmetry of microeconomic systems involving optimization, and 2) he was concerned with the political/normative implications such models (where the general equilibrium price vector corresponds to the maximization of something). As he said "there is the danger that unwarranted teleological and normative welfare significance will be attributed to a position of equilibrium so defined" (1947, p. 52) and that seems to be an obvious danger when the equilibrium p^* always maximizes some (welfare?) function. In his reflection "How Foundations Came to Be" Samuelson discusses "the road not taken" – the possibility of ending Foundations after Part I on optimization and neglecting dynamics entirely: "The result would have been a shorter 200-page book with one fully-integrated theme" (1998, p. 1384). He explains why he did not take this path:

> I could not resist the temptation to add Part Two on dynamics, even though much of my focus there was on "macroeconomics" (a word not yet coined …). No one associates a Keynesian system with a maximizing single mind or even to an as-if-pretend maximizing system. Yet from consideration of The General Theory's "stable" dynamics, one could predict that a rise in the propensity to invest would increase, not lower, underemployment equilibrium output and GNP. … In retrospect, I have never regretted not taking the road not taken. (ibid.)

The bottom line is that if Samuelson had not wanted to maintain a sharp distinction between optimization and dynamics – for whatever reason – he surely would have put the symmetric case to work in various applications (the mathematical properties are very convenient). The fact that he did not do so, provides additional evidence that maintaining separation between these two conceptual frameworks for mathematical economic theory was fundamental to the Foundations program. The later stability literature followed Samuelson's lead on this; in the postwar era optimization was one thing and stability was

[14] This case is quite similar to the Hotelling model discussed in detail in Hands and Mirowski (1998).

something else entirely. The symmetric/gradient case was mentioned from time to time throughout the stability literature – Arrow and Hahn (1971, pp. 275-78), Arrow and Hurwicz (1958, p. 535) and Varian (1982, p. 105) for example – but always in passing and always as a special case that does not solve the stability problem. As Arrow and Hahn put it: "Unfortunately, however, except in exceptional circumstances … the price mechanism cannot be taken to act as if someone were trying to maximize or minimize some well-behaved function of prices" (Arrow and Hahn, 1971, p. 278).

So the general story is that the stabilization of dynamics as separate from optimization removed the dynamic element form consumer choice theory and thereby stabilized what had been a diverse, but contentious, field of theoretical inquiry. If consumer choice did not involve movement, then there would be none of problems associated with time and position in consumer choice – the problems integrability$_B$ and the problems that have re-emerged in the recent literature. The fact is that consumer choice theory would never have stabilized around the budget-constrained utility-maximizing model (or any other model for that matter) if the buzzing controversy of the 1930s had continued – agreement is a precondition for normal science – and so stabilizing consumer choice required stabilizing integrability. Once it was agreed that dynamics meant having an explicit representation by differential or difference equations, then consumer choice was no longer dynamic and the various integrability problems conveniently went away.

It is important to note that this story says nothing about design. There is nothing in the story that suggests that it was the intention of Samuelson or anyone else to stabilize consumer choice theory. There is certainly no evidence to that effect; in the early 1940s there was no generally agreed upon consumer choice theory to defend and it is important to remember that in 1938 Samuelson was advocating his own weak axiom-based theory of consumer behavior. Samuelson's relationship to consumer choice theory is complex, and may have changed over time,[15] but consumer choice theory was just one chapter in, not the raison d'être for, Foundations (or for the strict separation of optimization from stability therein). Samuelson had many reasons for the theoretical framework he advocated – many of these were discussed above – and the other economists who adopted his framework all had their own reasons as well (sometimes the same, and sometimes quite different); the fact that consumer choice theory ended up stabilized in part as a result of these actions does not imply fulfillment of the intentions of the agents involved. Dominant forms systematically emerge from the bubbling diversity in science as well as within nature, and design is no more necessary in the former than the latter.

This story does though, seem to have rather ironic implications for the relationship between consumer choice theory and the stability literature in general equilibrium theory. It is, as discussed above, generally, and in a sense rightly, considered to be one of the "problems" of Walrasian theory . In Franklin Fisher's recent words "the lack of a fully satisfactory stability analysis is a gaping hole in microeconomic theory" (Fisher, 2006, p. 142). The irony is that consumer choice theory is at the core of general equilibrium

[15] Samuelson's relationship to consumer choice theory – particularly as it relates to revealed preference theory and integrability – is discussed in detail in Hands (2006a, 2007), but there is certainly much more to be said.

theory, and it seems very unlikely that Walrasian theory would have risen to the dominant position it had once had in the profession – at least looking anything like it did – without a generally accepted theory of consumer choice. The theoretical pluralism of the 1930s would not have served as the cornerstone for a tight formalized theoretical framework like the Arrow-Deberu model. Undoubtedly the two stabilized together, but as a matter of historical sequence consumer choice theory converged first and lived beyond general equilibrium theory's fall from grace. And yet, what essentially is the stability problem in general equilibrium theory? The problem is that <u>consumer</u> <u>choice</u> <u>theory</u> <u>does</u> <u>not</u> <u>put</u> <u>enough</u> <u>structure</u> <u>on</u> <u>demand</u> <u>functions</u> to guarantee stability (or uniqueness, or determinate comparative statics). The separation of dynamics from optimization, itself partly motivated by the attempt to capture Walrasian dynamics, helped stabilize consumer choice theory, and yet the main problem for stability theory – one of the key threads unraveling the Arrow-Debreu program – was the failure of consumer choice theory to put enough structure on aggregate excess demand to guarantee stability. An ironic turn of theoretical events indeed.

4. <u>Conclusion</u>

There is no single explanation for why/how mid-twentieth century microeconomic theory came to be what it came to be – why the diversity of the previous decades was replaced by a single dominant formalist research program based on rational economic agents, competitive markets, and general equilibrium. Philip Mirowski has documented the changes brought about by the evolving role of the physics metaphor (1989), the impact of the computer/computation and the imperatives of the Cold War (Mirowski 2002). Roy Weintraub has explained how the profession's desire to reconcile the unemployment and apparent disequilibrium of economic life during the 1930s with its commitment to the stability of the competitive market contributed to these changes (Weintraub 1991), and Weintraub and others have examined the impact of various intellectual developments within pure mathematics (Boylan and O'Gorman 2007, Weintraub 2002, Weintraub and Mirowski 1994). Elsewhere I have documented the role played by the narrowing of the integrability concept (Hands 2006a), the evolution of Samuelson's revealed preference program and the relationship between economics and psychology (Hands 2006b, 2007); and in joint research with Philip Mirowski how, once established, dominance was reinforced by inherent resiliency of the three-schools ecosystem that eventually stabilized within mid-twentieth century price theory (Hands and Mirowski 1998, Mirowski and Hands 1998). And these are only a few of the influences that have been discussed in the recent historical literature. Econometrics certainly mattered, as did the decline of institutionalism, the professionalization of the discipline, changing conceptions of the individual, Marxism both as an alternative economic theory and a political reality, changes in the dominant characterization of scientific knowledge and causality, and a host of other factors (Amadae 2003, Bernstein 2001, Davis 2003, Giocoli 2003, Morgan and Rutherford 1998, Yonay 1998 and others).

Here I have explored an aspect of the story that seems to have been previously unexamined: the role played by shifting theoretical discussion about dynamics and stability over to the competitive market – the inter-agent stability of the Walrasian tâtonnement – and away from the behavior of the individual consumer. Earlier the consumer had been conceived as fundamentally dynamic – consumer choice involved movement in space and over time – but this opened the door to the pernicious effects of time and a host (recently rediscovered) issues about path-dependency, endowment effects, framing, and such. By sharply demarcating the optimization-based equilibria of utility-maximizing consumers from the dynamic adjustments of the competitive market, and by shifting all dynamic considerations exclusively on to the latter – a move in which Samuelson's Foundations played a key, but not the only, role – consumer choice theory was insulated from the host of critical concerns that had earlier been associated with integrability$_B$. The one fixed point in all the controversies within economic theory during the period 1950-80s – from Keynesianism versus Monetarism, to Cambridge capital controversies, to the rise of Nash-based game theory, and all the rest – was the budget-constrained utility-maximizing model of consumer choice. Consumer choice theory became normal science to a degree, and over a period of time, unmatched by any other theoretical framework in the history of modern economic thought. It has started to show some signs of wear during the last decade or so – although it still remains firmly ensconced in textbooks at every level – but for much of the twentieth century consumer choice theory represented the crown jewel of economic science. My argument is simply that this would not have been the case had it not been possible to brush away the issues of time, position, and integrability$_B$; and that the rise of "true dynamic stability" played a significant role in the profession's ability to do just that. Whether one views this influence positively (as getting beyond the confusing muddle of the 1930s and providing the cornerstone for the discipline's success), or negatively (as the suppression of serious problems within rational choice theory), really doesn't matter to the story presented here. The fact is that the middle of the twentieth century was the heyday of abstract mathematical economic theory, and in such economic theory dynamic adjustment plays no role in consumer choice. This said, it is not clear where the profession will go from here. Is there in fact a sea-change underway that will ultimately extirpate the utility-maximizing consumer from the core of economic theory? That is certainly too early to call, but hopefully this paper has given the reader a better understanding of the various forces that helped shape the theory that is now (again) being debated.

References

Allen, R. G. D. (1932), "The Foundations of a Mathematical Theory of Exchange," Economica, 36, 197-226.

Allen, R. G. D. (1936), "Professor Slutsky's Theory of Consumer's Choice," Review of Economic Studies, 3, 120-29.

Allen, R. G. D. (1950), Mathematical Analysis for Economists. London: MacMillan & Co. [First published 1938].

Amadae, Sonja M. (2003), Rationalizing Capitalist Democracy: The Cold War Origins of Rational Choice Liberalism. Chicago: University of Chicago Press.

Arrow, Kenneth J., Block, H. David, and Hurwicz, Leonid (1959), "On the Stability of the Competitive Equilibrium II," Econometrica, 27, 82-109.

Arrow, Kenneth and Debreu, Gerard (1954), "Existence of an Equilibrium for a Competitive Economy," Econometrica, 22, 265-290.

Arrow, Kenneth and Enthoven, Alain C. (1961), "Quasi-Concave Programming," Econometrica, 29, 779-800.

Arrow, Kenneth J. and Hahn, Frank H. (1971), General Competitive Analysis. San Francisco: Holden-Day.

Arrow, Kenneth J. and Hurwicz, Leonid (1958), "On the Stability of the Competitive Equilibrium I," Econometrica, 26, 522-552.

Bernstein, Michael A. (2001), A Perilous Progress: Economics and Public Purpose in Twentieth-Century America. Princeton, NJ: Princeton University Press.

Boylan, Thomas A. and O'Gorman, Paschal F. (2007), "Axiomatization and Formalism in Economics," Journal of Economic Surveys, 21, 426-46.

Bushaw, Dale W. and Clower, Robert W. (1957), Introduction to Mathematical Economics. Homewood, IL: Richard D. Irwin.

Camerer, Colin F. and Loewenstein, George (2004), "Behavioral Economics: Past, Present, Future," in Advances in Behavioral Economics, C. F. Camerer, G. Loewenstein, and M. Rabin (eds.), Princeton: Princeton University Press, 3-51.

Chaing, Alpha C. (1984), Fundamental Methods of Mathematical Economics, 3rd Edition, New York: McGraw Hill [First edition 1967].

Davis, John B. (2003), The Theory of the Individual in Economics. London: Routledge.

Debreu, Gerard (1954), "Representation of a Preference Ordering by a Numerical Function," in Decision Processes, R. M. Thrall, C. H. Coombs, and R. L. Davis (eds.), New York: Wiley, 159-65.

Debreu, Gerard (1959), Theory of Value. New York: Wiley.

Debreu, Gerard (1974), "Excess Demand Functions." Journal of Mathematical Economics, 1, 15-21.

Evans, Griffith C. (1930), Mathematical Introduction to Economics. New York: McGraw Hill.

Fernandez-Grela, Manuel (2006), "Disaggregating the Components of the Hicks-Allen Composite Commodity," in Agreement on Demand: Consumer Theory in the Twentieth Century, P. Mirowski and D. W. Hands (eds.), Durham, NC: Duke University Press [Annual Supplement to Volume 38 of History of Political Economy], 32-47.

Fisher, Franklin (2006), "Paul Samuelson and the Stability of General Equilibrium," in Samuelsonian Economics and the Twenty-First Century. M. Szenberg, L. Ramrattan, and A. A. Gottesman (eds.), Oxford: Oxford University Press, 142-45.

Frisch, Ragnar (1971), "On a Problem of Pure Economics," in Preferences, Utility, and Demand, J. S. Chipman, L. Hurwicz, M. Richter, and H. F. Sonnenschein (eds.), New York: Harcourt Brace Javanovich, 386-423 [translated by J. S. Chipman, original published in 1926].

Gale, David (1963), "A Note on Global Instability of Competitive Equilibrium," Naval Research Logistics Quarterly, 10, 81-87.

Georgescu-Roegen, Nicholas (1936), "The Pure Theory of Consumer's Behaviour," Quarterly Journal of Economics, 50, 545-93.

Giocoli, Nicola (2003), Modeling Rational Agents: From Interwar Economics to Early Modern Game Theory. Cheltenham, UK: Edward Elgar.

Hahn, Frank (1982), "Stability," in Handbook of Mathematical Economics, Vol. II, Amsterdam, NL: North-Holland, 745-793.

Hands, D. Wade (1994), "Restabilizing Dynamics," Economics and Philosophy, 10, 243-83.

Hands, D. Wade (2006a), "Integrability, Rationalizability, and Path-Dependency in the History of Demand Theory," in Agreement on Demand: Consumer Theory in the

Twentieth Century, P. Mirowski and D. W. Hands (eds.), Durham, NC: Duke University Press [Annual Supplement to Volume 38 of History of Political Economy], 153-85.

Hands, D. Wade (2006b), "Individual Psychology, Rational Choice, and Demand: Some Remarks on Three Recent Studies," Revue de Philosophie Economique, 13, 3-48.

Hands, D. Wade (2007), "Economics, Psychology, and the History of Consumer Choice Theory" Available at SSRN: http://ssrn.com/abstract=988125

Hands, D. Wade and Mirowski, Philip (1998), "Harold Hotelling and the Neoclassical Dream," in Economics and Methodology: Crossing Boundaries, R. Backhouse, D. Hausman, U. Mäki, and A. Salanti (eds.), London: Macmillan, 322-397.

Henderson, James M. and Quandt, Richard E. (1958), Microeconomic Theory: A Mathematical Approach. New York: McGraw Hill.

Hicks, John R. (1946), Value and Capital, 2nd Edition, Oxford: Clarendon Press [first edition 1939].

Hicks, John R. and Allen, R. G. D. (1934), "A Reconsideration of the Theory of Value, Parts I and II," Economica, 2, 52-76, 196-219.

Ikeo, Aiko (1994), "When Economics Harmonized Mathematics in Japan: A History of Stability Analysys," European Journal of the History of Economic Thought, 1, 577-599.

Ingrao, Bruna and Israel, Giorgio (1990), The Invisible Hand: Economic Theory in the History of Science. Cambridge, MA: MIT Press.

Kahneman, Daniel (2003), "Maps of Bounded Rationality: Psychology for Behavioral Economics," American Economic Review, 93, 1449-75.

Kahneman, Daniel and Tversky, Amos (eds.)(2000), Choices, Values, and Frames. Cambridge: Cambridge University Press.

Kirman, Alan (1989), "The Intrinsic Limits of Modern Equilibrium Theory: The Emperor Has No Clothes," The Economic Journal, 99, 126-39.

Kirman, Alan (2006), "Demand Theory and General Equilibrium: From Explanation to Introspection, A Journey Down the Wrong Road," in Agreement on Demand: Consumer Theory in the Twentieth Century, P. Mirowski and D. W. Hands (eds.), Durham, NC: Duke University Press [Annual Supplement to Volume 38 of History of Political Economy], 246-80.

Knetsch, Jack L. (1989), "The Endowment Effect and Evidence of Nonreversible Indifference Curves," The American Economic Review, 79, 1277-1284.

Lange, Oscar (1944), Price Flexibility and Employment. Bloomington, IN: Principia Press.

Mantel, Rolf R. (1974), "On the Characterization of Aggregate Excess Demand," Journal of Economic Theory, 7, 348-353.

Mantel, Rolf R. (1977), "Implications of Microeconomic Theory for Community Excess Demand Functions," in Frontiers in Quantitative Economic IIIA, M. D. Intriligator (ed.), Amsterdam: North-Holland, 111-26.

Mas-Colell, Andreu; Whinston, Michael D.; and Green, Jerry R. (1995), Microeconomic Theory. New York: Oxford University Press.

McKenzie, Lionel W. (1960), "Stability of Equilibrium and the Value of Positive Excess Demand," Econometrica, 28, 606-17

McKenzie, Lionel W. (2002), Classical General Equilibrium Theory. Cambridge: Cambridge University Press.

Metzler, Lloyd (1945), "Stability of Multiple Markets: The Hicks Conditions," Econometrica, 13, 277-92.

Mirowski, Philip (1989), More Heat than Light: Economics as Social Physics: Physics as Nature's Economics. Cambridge: Cambridge University Press.

Mirowski, Philip (2002), Machine Dreams: Economics Becomes a Cyborg Science. Cambridge: Cambridge University Press.

Mirowski, Philip and Hands, D. Wade (1998), "A Paradox of Budgets: The Postwar Stabilization of American Neoclassical Demand Theory," in From Interwar Pluralism to Postwar Neoclassicism, M. S. Morgan and M. Rutherford (eds.), Durham, NC: Duke University Press [Annual Supplement to Volume 30 of History of Political Economy], 269-92.

Mirowski, Philip and Hands, D. Wade (eds.)(2006), Agreement on Demand: Consumer Theory in the Twentieth Century, Durham, NC: Duke University Press [Annual Supplement to Volume 38 of History of Political Economy].

Morgan, Mary S. and Rutherford, Malcolm (eds.)(1998), Interwar Pluralism to Postwar Neoclassicism, Durham, NC: Duke University Press [Annual Supplement to Volume 30 of History of Political Economy].

Mosak, Jacob l. (1944), General Equilibrium Theory in International Trade. Bloomington, IN: Principia Press.

Moscati, Ivan (2007), “Early Experiments in Consumer Demand Theory: 1930-1970,” History of Political Economy, 39, 359-401.

Mukherji, Anjan (1973), “On the Sensitivity of Stability Results to the Choice of Numeraire,” Review of Economic Studies, 40, 427-33.

Newman, Peter (1959), "Some Notes on Stability Conditions," Review of Economic Studies, 26, 1-9.

Newman, Peter (1961), "Approaches to Stability Analysis," Economica, 28, 12-29.

Negishi, Takashi (1962), "The Stability of a Competitive Economy: A Survey Article," Econometrica, 30, 635-69.

Pareto, Vilfredo (1971), Manual of Political Economy. New York: Agustus M. Kelley [translated by Ann S. Schwier from the 1927 French edition].

Rabin, Matthew (1998), "Psychology and Economics," Journal of Economic Literature, 36, 11-46.

Rabin, Matthew (2004), “Behavioral Economics,” in New Frontiers in Economics, M. Szenberg and L. Ramrattan (eds.), Cambridge: Cambridge University Press, 68-102.

Rizvi, S. Abu Turab (2006), “The Sonnenschein-Mantel-Debreu Results After Thirty Years," in Agreement on Demand: Consumer Theory in the Twentieth Century, P. Mirowski and D. W. Hands (eds.), Durham, NC: Duke University Press [Annual Supplement to Volume 38 of History of Political Economy], 228-45.

Samuelson, Paul A. (1938), "A Note on the Pure Theory of Consumer's Behaviour," Economica, 5, 61-71.

Samuelson, Paul A. (1941), “The Stability of Equilibrium: Comparative Statics and Dynamics.” Econometrica, 9, 97-120.

Samuelson, Paul A. (1942), “The Stability of Equilibrium: Linear and Non-Linear Systems,” Econometrica, 10, 1-25.

Samuelson, Paul A. (1944), “The Relation Between Hicksian Stability and True Dynamic Stability,” Econometrica, 12, 256-257.

Samuelson, Paul A. (1947), Foundations of Economic Analysis. Cambridge, MA: Harvard University Press.

Samuelson, Paul A. (1950), "The Problem of Integrability in Utility Theory," Economica, 17, 355-85.

Samuelson, Paul A. (1998), "How Foundations Came to Be," Journal of Economic Literature, 36, 1375-1386.

Scarf, Herbert (1960), "Some Examples of Global Instability of the Competitive Equilibrium," International Economic Review, 1, 157-72.

Schultz, Henry (1938), The Theory and Measurement of Demand, Chicago: University of Chicago Press.

Shafer, Wayne and Sonnenschein, Hugo (1982), "Market Excess Demand Functions," in Handbook of Mathematical Economics, Vol. II, K. J. Arrow and M. D. Intriligator (eds.), Amsterdam: North-Holland, 671-93.

Slutsky, E. E. (1915), "Sulla Teoria del Bilancio del Consonatore," Giornale degli Economisti, 51, 1-26. Translated in Readings in Price Theory, G. J. Stigler and K. E. Boulding (eds.), Homewood, IL: Richard D. Irwin, 1952, 27-56.

Sonnenschein, Hugo (1972), "Market Excess Demand Functions," Econometrica, 40, 549-63.

Sonnenschein, Hugo (1973), "Do Walras' Identity and Continuity Characterize the Class of Community Excess Demand Functions?," Journal of Economic Theory, 6, 345-54.

Takayama, Akira (1974), Mathematical Economics. Hinsdale, IL: The Dryden Press.

Thaler, Richard H. (1980), "Toward a Positive Theory of Consumer Choice," Journal of Economic Behavior and Organization, 1, 39-60 [reprinted as Chapter 15 of Kahneman and Tversky 2000].

Uzawa, Hirofumi (1961), "The Stability of Dynamic Processes," Econometrica, 29, 617-31.

Varian, Hal R. (1982), "Dynamical Systems with Applications to Economics," in Handbook of Mathematical Economics, Vol. I, Amsterdam, NL: North-Holland, 93-110.

Weintraub, E. Roy (1991), Stabilizing Dynamics: Constructing Economic Knowledge. Cambridge: Cambridge University Press.

Weintraub, E. Roy (2002), How Economics Became a Mathematical Science. Durhan, NC: Duke University Press.

Weintraub, E. Roy and Mirowski, Philip (1994), "The Pure and Applied: Bourbakism Comes to Mathematical Economics," Science in Context, 7, 245-72.

Yonay, Yuval (1998), The Struggle Over the Soul of Economics. Princeton, NJ: Princeton University Press.